Phase Transitions and Volunteering in Spatial Public Goods Games

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We present a simple yet effective mechanism promoting cooperation under full anonymity by allowing for voluntary participation in public goods games. This natural extension leads to "rock-scissors-paper"–type cyclic dominance of the three strategies, cooperate, defect, and loner. In spatial settings with players arranged on a regular lattice, this results in interesting dynamical properties and intriguing spatiotemporal patterns. In particular, variations of the value of the public good leads to transitions between one-, two-, and three-strategy states which either are in the class of directed percolation or show interesting analogies to Ising-type models. Although volunteering is incapable of stabilizing cooperation, it efficiently prevents successful spreading of selfish behavior.

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In behavioral sciences and more recently in economics the evolution of cooperation among unrelated individuals represents one of the most stunning phenomena [1,2]. The prisoner's dilemma (PD) has long been established as a paradigm to explain cooperative behavior through pairwise interactions [3]. While the PD attracted attention from biologists and social scientists, most studies in experimental economics focused on the closely related but more general public goods game (PGG) for group interactions [4]. In typical PGG experiments, an experimenter endows, e.g., four players with \$10 each. The players then have the opportunity to invest part or all of their money into a common pool. They know that the total amount in the pool is doubled and equally divided among all participants irrespective of their contributions. If everybody cooperates and contributes their money, each player ends up with \$20. However, every player faces the temptation to defect and to free ride on the other players' contributions by withholding the money since every invested dollar returns only 50 cents to the investor. Obviously, defection represents the dominating strategy leading to the "rational" equilibrium where no one increases the initial capital. Such strategical behavior prescribed to *homo oeconomicus* is frequently at odds with experimental findings [5] and lead to the decline of this rationality concept.

Note that for pairwise encounters with a fixed investment amount, the PGG reduces to the PD. PGG interactions are abundant in animal and human societies [6–8]. Consider, for example, predator inspection behavior, alarm calls, and group defense, as well as health insurance, public transportation, or environmental issues, to name only a few.

Recently it was demonstrated that voluntary participation in such public enterprises may provide an escape hatch out of economic stalemate and results in a substantial and persistent willingness to cooperate even in sizable groups, in the absence of repeated interactions, under full anonymity, and without secondary mechanisms such as punishment or reward [9].

The voluntary participation in the PGG is modeled by considering three strategical types of players: (i) cooperators C and (ii) defectors D, both willing to join the PGG, though with different intentions. While the former are ready to contribute a fixed share to the common pool, the latter attempt to exploit the resource. Finally there are the so-called (iii) loners L, who refuse to participate and rather rely on some small but fixed income. The loner strategy is thus risk averse. These strategies lead to a "rock-scissors-paper" dynamics with cyclic dominance: if cooperators abound, they can be exploited by defectors; if defectors prevail, it is best to abstain; and if no one participates in the PGG, small groups can form and it pays to return to cooperation. Therefore, voluntary participation provides a simple yet natural way to avoid deadlocks in states of mutual defection. In well-mixed populations, i.e., in mean-field type models with replicator dynamics [10], this system can be solved analytically [11].

In this Letter, we consider a spatially extended variant of the voluntary PGG where players are arranged on a rigid regular lattice and interact only with their local neighborhood. Each player is confined to a site $\mathbf{x}$ on a square lattice. The size of the neighborhood therefore determines the maximum number of participants N in the PGG. We restrict our investigations to the *von Neumann* neighborhood, i.e., to $N = 5$. But note that the qualitative results remain unaffected by the underlying geometry of the regular lattice. The state variable $s(\mathbf{x}) \in \{C, D, L\}$ determines the player's strategy at any given time. The score achieved in PGG interactions denotes the reproductive success, i.e., the probability that one of the neighbors will adopt the player's strategy. In the rigorous sense of the spatial PGG, this score is accumulated over $N = 5$ games, i.e., by summing up the player's performance in PGGs taking place on the player's site as well as on the neighboring sites. For the sake of simplicity, we assume that the score $P(\mathbf{x})$ is determined by a single, typical PGG involving the player and its four nearest neighbors. This simplification accelerates the simulations and makes the pair approximation more convenient while causing minor modifications in the system's dynamics.

The score $P(\mathbf{x})$ depends on the five strategies. Namely, if n_c, n_d, and n_l (with $n_c + n_d + n_l = N = 5$) denote the number of participants choosing C, D, and L, then

$$P(\mathbf{x}) = \begin{cases} \frac{rn_c}{n_c+n_d} - 1 & \text{if } s(\mathbf{x}) = C, \\ \frac{rn_c}{n_c+n_d} & \text{if } s(\mathbf{x}) = D, \\ \sigma & \text{if } s(\mathbf{x}) = L, \end{cases} \tag{1}$$

where the cooperative investments are normalized to unity and r specifies the multiplication factor on the public good. Note that $r > 1$ must hold such that groups of cooperators are better off than groups of defectors—hence to establish a social dilemma. The loner payoff σ with $0 < \sigma < r - 1$ denotes a small but reliable source of income with a lower performance than mutual cooperation but better than mutual defection. Solitary C or D players ($n_c + n_d = 1$) are assumed to act as loners.

Players reassessing and updating their strategies are randomly chosen (e.g., at site $\mathbf{x}$) and compare their score to a randomly chosen neighbor $\mathbf{y}$. $\mathbf{x}$ adopts the strategy of $\mathbf{y}$ with a probability [12]:

$$W[s(\mathbf{y}) \to s(\mathbf{x})] = \frac{1}{1 + \exp\{[P(\mathbf{x}) - P(\mathbf{y}) + \tau]/K\}}, \tag{2}$$

where $\tau > 0$ denotes the cost of strategy change and K introduces some noise to allow for irrational, i.e., non-payoff-maximizing choices. For $K = 0$ the neighboring strategy $s(\mathbf{y})$ is always adopted provided the payoff difference exceeds the cost of strategy change, i.e., $P(\mathbf{y}) > P(\mathbf{x}) + \tau$. For $K > 0$, strategies performing worse are also adopted with a certain probability, e.g., due to imperfect information. K determines the half-width of this probability distribution.

By means of Monte Carlo (MC) simulations complemented by pair approximation, we determine the equilibrium frequencies of the three strategies when varying r while keeping σ, K, and τ fixed. For the pair approximation we determine analytically the doublet density, i.e., the probability of all configurations of two neighboring sites [12]. Through moment closure, i.e., by approximating higher order densities (e.g., triplets) with doublet densities, a set of equations of motion is obtained which is solved numerically.

Qualitatively the dynamics remains unaffected when changing σ, K, and τ within realistic limits. Henceforth, we thus concentrate on the general features of spatiotemporal patterns and transitions. As we shall see, the cyclic dominance of the strategies acts as a driving force for traveling waves and leads to persistent and robust coexistence of all three strategies over a wide parameter range. Similar results have been found for an externally driven variant of the spatially extended PD with three strategies [13] or if sites are allowed to go empty [14]. The simulations are performed under periodic boundary conditions on an $M \times M$ lattice with $400 \leq M \leq 2000$.

Let us first briefly consider the compulsory PGG, i.e., with C and D only. The spatial extension may enable cooperators to persist by forming clusters and thereby minimizing exploitation by defectors. This is a well-known result from other cooperation games [12,15,16]. For sufficiently high $r > r_C$ cooperators survive with frequencies quickly increasing with r because C is favored for an increasing number of local configurations. In contrast, below the threshold r_S the system eventually reaches the homogeneous D state (see Fig. 1). Henceforth, the subscript α of r_α refers to the vanishing strategy.

In the close vicinity of r_C, the visualization of strategy distribution shows isolated colonies of C. These colonies move randomly and can coalesce or divide. Consequently, this system becomes equivalent to a branching and annihilating random walk [17] which exhibits a transition belonging to the directed percolation (DP) universality class [12,18–20]. According to MC simulations for $r \to r_C$ from above, the frequency of C is proportional to $(r - r_C)^\beta$ with $r_C = 4.526(1)$ and $\beta = 0.55(3)$ for $\sigma = 1$ and $K = \tau = 0.1$. The pair approximation predicts a significantly lower critical value $r_C^{(p)} = 2.694$. This difference refers to the enhanced role of n-point ($n > 2$) correlations.

In the case of voluntary participation, the loners induce significant changes most pronounced at low r. The resulting dynamics can be divided into three regimes (see Fig. 2): (a) For $r < r_D = 1 + \sigma$ it is trivial that only loners survive since they perform better than groups of cooperators. Note that solitary C and D are eliminated by noise. (b) For

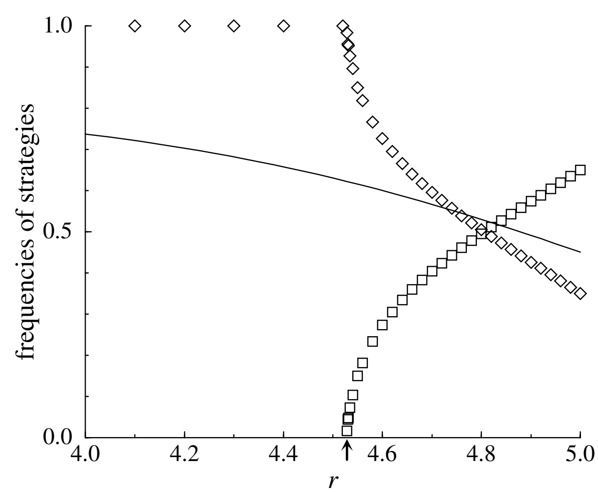

FIG. 1. Frequencies of cooperators C (open squares) and defectors D (open diamonds) as a function of the multiplication rate r for $\sigma = 1$ and $\tau = K = 0.1$. The solid line shows the frequency of defectors in pair approximation. The arrow indicates r_C where cooperators vanish.

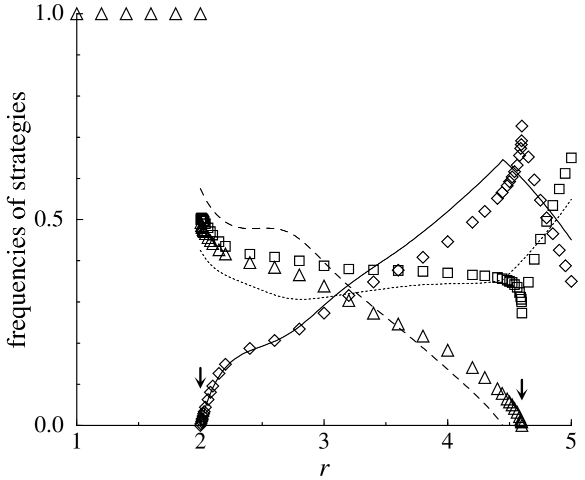

FIG. 2. Frequencies of cooperators C (open squares), defectors D (open diamonds), and loners L (open triangles) as a function of r for $\sigma = 1$ and $\tau = K = 0.1$. The results of pair approximation are shown as dotted (C), solid (D), and dashed (L) lines. The values of r_D and r_L are indicated by arrows.

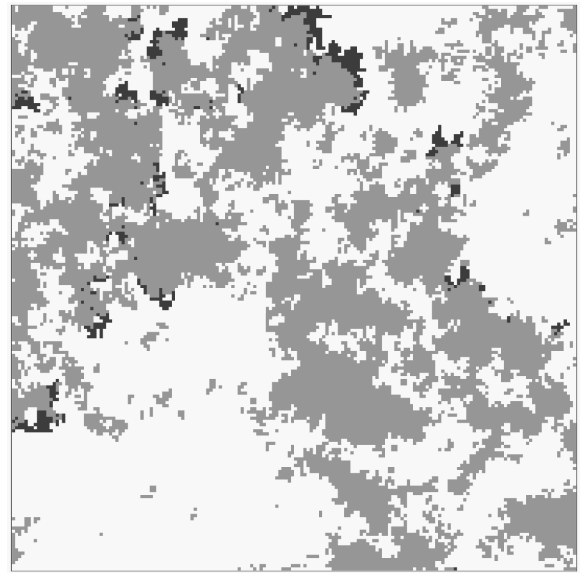

FIG. 3. Distribution of cooperators (white), defectors (black), and loners (gray) on a 200×200 portion of a larger lattice. The parameters are $\sigma = 1$, $\tau = K = 0.1$, and $r = 2.035$, i.e., slightly above the transition to homogeneous states of loners.

$r_D < r < r_L$ the three strategies coexist and produce fascinating spatiotemporal patterns including traveling waves. Such values of r almost invariably result in homogeneous D states in the compulsory PGG. Thus, the loners provide vital protection to cooperators against exploitation. (c) For $r > r_L$ cooperators again thrive on their own as in the compulsory PGG. Loners go extinct because they no longer provide a valuable alternative.

In the remaining text we discuss the coexistence regime in greater detail. According to our numerical analysis, the extinction of loners for $r \to r_L$ also exhibits a DP transition [19,21,22]. The frequency of L is proportional to $(r_L - r)^\beta$ in the vicinity of $r_L = 4.6005(5)$ with $\beta = 0.58(3)$ in agreement with previous data [20]. The increase of fluctuations in the frequency of loners is consistent with a power law divergence predicted by the scaling hypothesis [21,22].

The robustness of DP transitions is well demonstrated by noting that the two critical transitions belong to the same universality class despite remarkable differences. The extinction of C, leaving a homogeneous D state behind, contrasts with the extinction of L on a time-dependent, inhomogeneous $C + D$ background. Field-theoretic arguments indicate that the main features of DP remain unchanged if the spatiotemporal fluctuations of the random environment are uncorrelated [22]. Our numerical results support this expectation.

In the region of coexistence, the frequency of C remains within narrow limits compared to the trends observed for D and L. Figure 2 indicates that the pair approximation yields a suitable quantitative description. In particular, D vanishes linearly with $r \to r_D$. This behavior is strongly related to pattern evolution observed for low D frequency (see Fig. 3). The D strategy forms small black islands invading the territory of C. At the same time, defectors are in turn invaded by loners paving the way for the return of cooperators. The cyclic dominance maintains this self-organizing pattern. But defectors can easily die out if the system size is not large enough. The occasional extinction of D results in a homogeneous C state. Therefore, this requires extremely large system sizes and a careful preparation of the initial state. Interestingly, the fluctuations of the D frequency remains constant while the frequency itself vanishes linearly (see Fig. 4).

The typical domain size increases when r goes to r_D. Figure 4 illustrates that this is accompanied by a power law

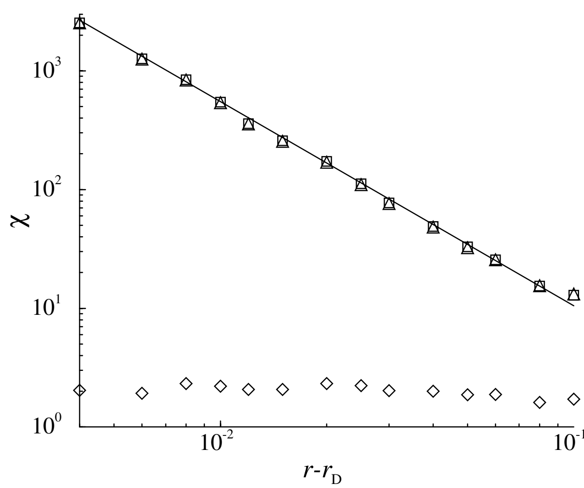

FIG. 4. Log-log plot of the frequency fluctuations χ vs $r - r_D$ (symbols and parameters as in Fig. 2). χ denotes the square of average fluctuation amplitudes produced by the system of size M^2. The solid line shows the fitted power law with $\gamma = 1.72$.

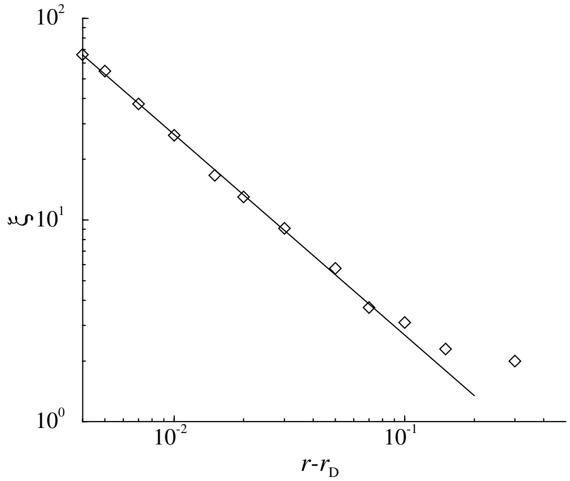

FIG. 5. Log-log plot of the correlation length ξ vs $r - r_D$ for the three strategies (parameters as in Fig. 2). The solid line denotes the fitted power law with an exponent -0.99.

divergence of the frequency fluctuations for C and L strategies $\chi_c \simeq \chi_l \propto (r - r_D)^{-\gamma}$. The numerical fit gives an exponent close to $\gamma = 7/4$ which is characteristic to the order parameter fluctuations in the Ising model when approaching the critical point from above [23]. One might argue that the multiplication factor r is related to an external field stimulating cooperation and the noise term K to temperature; however, a direct mapping seems impossible due to the additional dependence on the loner's payoff σ.

Another curiosity of this model refers to the equal frequencies of C and L in this limit. Moreover, the correlation length ξ (derived from the density-density correlation function; see Fig. 5) appears to be proportional to $1/(r - r_D)$. The formation of larger and larger domains in the two-dimensional, zero-field Ising model exhibits similar behavior when decreasing the temperature to the critical point [23]. This suggests that the universality of this Ising-type transition determines constraints on the size distributions of C and L domains. In this case the defectors with vanishing frequency contribute to maintain suitable domain dynamics.

To conclude, we introduced a spatial evolutionary PGG model demonstrating that the successful spreading of selfish behavior is efficiently prevented by allowing for voluntary participation. In the compulsory PGG, i.e., in the absence of loners, cooperators thrive only if clustering advantages are strong enough, which requires sufficiently high multiplication factors r. The introduction of loners leads to a cyclic dominance of the strategies and promotes substantial levels of cooperation where otherwise defectors dominate.

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