Coevolution of strategies and interactions in donation games on a square lattice

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The coevolution of strategies and interactions is investigated with players located on a square lattice with interactions between first and second neighbors. The pair interactions are described by two-strategy (cooperation or defection) donation games with variable cost and benefit parameters for each player. During the evolutionary process, the players can adopt strategy or payoff parameters from one of their neighbors with a payoff-dependent probability. Numerical simulations indicate the formation of homogeneous defection or cooperation depending on the initial strategy and payoff distributions. The introduction of weak and rare mutations in the payoff adoption proves beneficial for the cooperative state, in which it increases the average payoff linearly in time. In contrast, the population cannot capitalize on the possibility of innovations in its defective state.

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I. INTRODUCTION

During the last four decades, evolutionary game theory has become a general mathematical framework for the quantitative analysis of phenomena and relationships occurring in biological, ecological, and social systems, including human behavior [1–7]. Typically, game theory [8] defines the interaction between the players via payoff matrices indexed by a finite number of strategies representing different species or individual behaviors. The application of the methods of statistical physics to game-theoretic models [9–15] was initiated by the introduction of spatial games [16–18], in which the players are located at the sites of a lattice or a graph and the interactions are limited to connected players (neighbors). In traditional evolutionary games, the dynamical rules affect the evolution of strategy distributions. So-called coevolutionary models generalize this setup by allowing other relevant ingredients of the mathematical model to evolve too, such as the connectivity structure [19], the motion of individuals, the noise in imitation dynamics, or the reputation or other personal features of the players. (For a brief survey, see Refs. [15,20].)

In the present paper, the previous investigations of the spatial models are extended by allowing the evolution of the interactions themselves via imitation and mutation, which can represent innovation both in human and ecological systems. The evolution of payoff matrices has already been studied within the framework of the diploid population model [21] and when the environment reacts to the actions of the players [22–24]. Very recently, the transitions of 2×2 games on networks [25,26] and the stability of evolving two-strategy payoff matrices (in well-mixed populations) [27,28] were also investigated. Some aspects of the evolution of interactions can also be recognized in previous investigations in which the players use stochastic reactive strategies [29–32] in repeated two-player games. The possibility of continuous behaviors initiated many other investigations where payoff variations arise as a result of the evolution of mixed strategies [33–39]. The effects of game transitions were also investigated for the donation game under several types of dynamics [34,40–42].

One of the most intriguing problems in realistic multiagent modeling is raised by the fact that the players can interact with their neighbors via different two-player games using distinct strategies against their coplayers.

Here, we study a simple spatial coevolutionary donation game, in which the interactions between the neighboring players are described by donation games with parameters that depend on the properties of both participating players as described in the next section. Using this mathematical framework, we focus our efforts on investigating the variation of interactions when the players can stochastically adopt not only the strategy of a more successful neighbor but also their payoff parameters if they use the same strategy. This simple model is able to clarify several mechanisms that can suppress or, conversely, exacerbate social dilemma situations. The most relevant of these observations concerns the evolution of payoff components within homogeneous strategy domains. While we expect similar processes to play a similarly important role in systems involving more general interactions too, it also highlights how heavily mechanisms that support cooperation can rely on the specific details of the underlying social dilemma.

II. THE MODEL

We study a coevolutionary game in which the players are located at the sites $\mathbf{x} = (i, j)$ $(0 \leqslant i, j < L)$ of a square lattice under periodic boundary conditions. The players derive income from donation games played with their eight (nearest and next-nearest) neighbors. Each player chooses one of two strategies, denoted by $s(\mathbf{x}) = 0$ for defection (D) and $s(\mathbf{x}) = 1$ for cooperation (C). In the conventional donation game [6], the cooperative players pay a cost ($c > 0$) that yields a benefit ($b > 0$) to the coplayer. In the present model, each player has their own cost $[c(\mathbf{x})]$ and benefit $[b(\mathbf{x})]$ parameters. We assume that the players use the same pure strategy against all of their eight nearest neighbors. Thus, the accumulated

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income $U(\mathbf{x})$ of the player located at site $\mathbf{x}$ takes the form

$$U(\mathbf{x}) = \sum_{\delta\mathbf{x}} P_x(\mathbf{y}), \tag{1}$$

where the summation runs over the neighboring sites $\mathbf{y} = \mathbf{x} + \delta\mathbf{x}$, and

$$P_x(\mathbf{y}) = s(\mathbf{y})[b(\mathbf{x}) + b(\mathbf{y})]/2 - s(\mathbf{x})[c(\mathbf{x}) + c(\mathbf{y})]/2 \tag{2}$$

denotes the contribution from a donation game in which the cost and benefit parameters are determined by the average values of the interacting players. The strategy distribution evolves via consecutive strategy adoptions: A randomly chosen player (located at site $\mathbf{x}$) switches to the strategy of one of their neighbors (located at a uniformly randomly chosen neighboring site $\mathbf{y}$) with probability

$$w[s(\mathbf{x}) \leftarrow s(\mathbf{y})] = \frac{1}{1 + e^{-[U(\mathbf{y}) - U(\mathbf{x})]/K}}, \tag{3}$$

where K quantifies the noise affecting the update process [43]. This strategy imitation process only actually modifies the strategy distribution when $s(\mathbf{x}) \neq s(\mathbf{y})$. What makes our model different lies in the extension of this elementary process by the adoption of payoff parameters with the same probability as above when $s(\mathbf{x}) = s(\mathbf{y})$. During this latter imitation process, we also allow some mutations to occur in the payoff parameters. Namely, the adopted payoff parameters are modified by a small value of $\delta c = \pm 0.01$ and $\delta b = \pm 0.01$ with rate r. Assuming that $100c(\mathbf{x})$ and $100b(\mathbf{x})$ are integers helps alleviate some technical difficulties in the numerical simulations.

We measure time in units of Monte Carlo steps (MCSs), the amount of time it takes for the evolutionary process to allow each player one chance on average to modify their strategy or payoff parameters.

We choose the eight-player Moore-interaction neighborhood—rather than the four-player von Neumann neighborhood, which is more prevalent in the literature of spatial donation games—in an attempt to better approximate real systems, which are typically characterized by larger average degrees and nonzero clustering coefficients.

For reference, Fig. 1 shows the average frequency of cooperators as a function of the cost parameter for various noise levels when the payoff parameters are uniform and fixed with $b = 1$. In this simpler, conventional evolutionary game, the two strategies coexist when $c_1(K) < c < c_2(K)$. The existence of D strategies at negative c values indicates spiteful behavior [44], when the defectors give up some income in order to reduce the income of their opponent, thereby increasing the probability that their strategy is imitated. Population dynamics (mean-field approximation) correctly predicts behavior in the $K \to \infty$ limit, when $c_1(K)$ and $c_2(K) \to 0$. In agreement with theoretical expectations, the extinction of the C strategy (as $c \to c_2$ from below) clearly exhibits the general features of critical phase transitions belonging to the directed percolation universality class [45]. The extinction of the D strategy (as $c \to c_1$ from above) also follows a similar power law in simulations performed on large systems ($L > 1000$) with long thermalization and sampling times ($\tau_t, \tau_s > 10^5$ MCS).

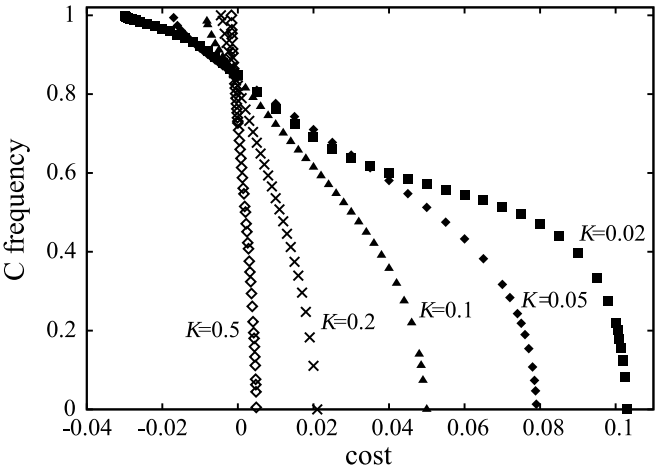

FIG. 1. Average frequency of cooperators versus c on the square lattice with nearest- and next-nearest-neighbor interactions for several values of the noise level K when the payoff parameters are uniform and fixed ($b = 1$).

Previously, the c dependence of cooperator frequency was found to be similar when the interactions are limited to the four nearest neighbors. (See Ref. [10] and references therein.) The most striking difference is that the strategies can coexist in a wider region of c values when the players are allowed to interact with eight neighbors.

III. COEVOLUTIONARY PROCESSES

When payoff parameter variation is introduced into the model, the two homogeneous states of all players choosing either cooperation or defection remain absorbing states in the space of strategy distributions. The evolution of the payoff parameters, however, keeps occurring within these absorbing states and exhibits significantly different behaviors.

Figure 2 shows how the average benefit evolves when all players cooperate for two different initial states in a large

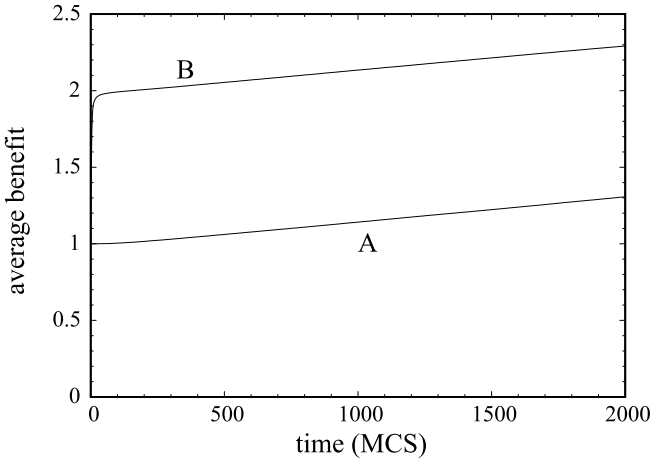

FIG. 2. Average benefit versus time when all players cooperate in the initial state for $L = 1000$, $K = 0.1$, and $r = 0.01$. Curve A shows the MC results when $b(\mathbf{x}) = 1$ and $c(\mathbf{x}) = 0.1$ in the initial state. Curve B indicates results in a system whose initial benefit and cost values were selected randomly from within these regions: $0 < b(\mathbf{x}) < 2$ and $0 < c(\mathbf{x}) < 0.2$.

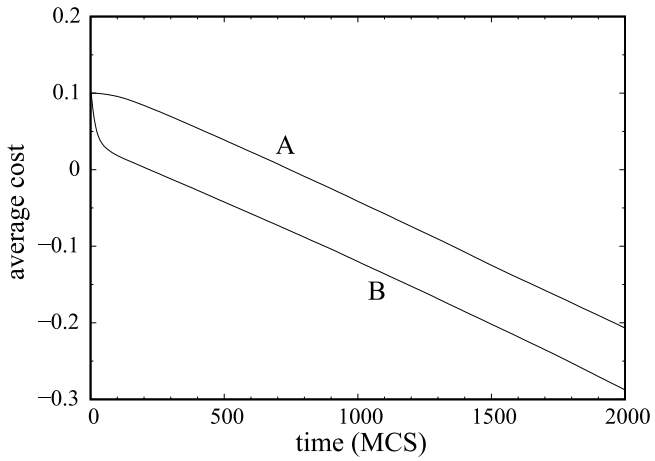

FIG. 3. Average cost versus time when all players cooperate in the initial state. Curves A and B correspond to the same parameters of the dynamical rules and initial states as those given in Fig. 2.

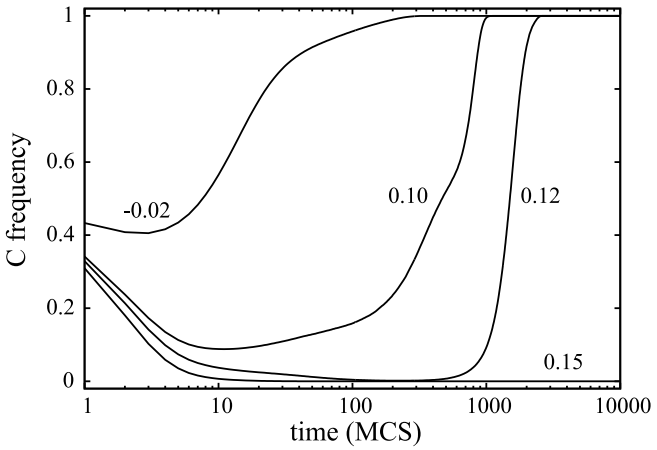

FIG. 4. Time-dependence of C frequency for homogeneous initial benefit ($b = 1$) and cost (indicated by labels) values at a low noise level ($K = 0.02$) and slow mutations ($r = 0.01$, $\delta c, \delta b = \pm 0.01$) with randomly assigned initial C and D strategies.

system at a low noise level. The average values of the payoff parameters are equal in the initial states in the two simulations, but soon become significantly different.

Mutations evolve homogeneous payoff parameter distributions in such a way that yields a permanent increase in the average benefit. The velocity of this increase depends on the noise parameter K and varies linearly with δb and r. When the initial payoff parameter distribution is inhomogeneous, the average benefit grows much faster within a short transient period ($t < 100$ MCS) as the payoff adoption process spreads the highest benefits across the system. Subsequently, the rate of increase of the average benefit matches the one we found for the corresponding homogeneous initial distribution. The boundless growth of the average benefit can be curbed by imposing an upper limit, which often arises quite naturally in concretized biological or economic examples. In our model, this would result in the benefit saturating at its maximal allowed value.

The evolution of the average cost exhibits similar behavior, as shown in Fig. 3; the smaller distance between the asymptotic decreases of the average values results from the narrower region of cost parameters present in the initial state.

The equivalence between the average decrease of c and the average increase of b in the results presented in the two figures is not an inherent general feature of the model but a direct consequence of specific parameter choices, which follows from the interchangeability of the roles of b and $-c$ in Eq. (2) and the mutation rates of the cost and benefit parameters being identical.

The most important consequence of the consecutive variations of payoff parameters is that the "donation game" transforms into the "harmony game," in which $c < 0$ and mutual cooperation is the only Nash equilibrium.

When the initial strategy distribution is homogeneous defection, payoff evolution takes a significantly different course. Using the same payoff parametrization as before [see Eq. (2)], all defectors receive zero income, which leads to random payoff parameter adoptions between the neighbors. This stochastic process does not modify the average values of either the benefits or the costs, while the mutations increase their standard deviations monotonically.

The features outlined above are preserved inside sufficiently large homogeneous domains of cooperators or defectors. Thus, the coevolutionary payoff parameter update rules indirectly affect the competition of defection and cooperation in spatial systems. For small system sizes (e.g., $L < 100$), sufficiently large C domains occur rarely, so the system can become fixed in a state of homogeneous defection. To reduce this finite-size effect, we used large system sizes ($L = 3000$) to explore how the coevolutionary processes support the maintenance of cooperation.

Our results presented in Fig. 4 indicate that the spiteful behavior [44] observed for negative cost values in the conventional model (see Fig. 1) is suppressed. Cooperative behavior is similarly supported for small cost parameters at low noise levels. The system evolves into homogeneous cooperation even for $c(\mathbf{x}) = 0.12$, when defection would dominate the stationary state in the conventional model. The fourth curve in Fig. 4 shows an example of the system falling into the trap of social dilemma due to high initial costs.

In Fig. 5, the MC simulation results indicate similar outcomes for high noise levels in line with mean-field-type behavior. Such spatial interactions ($c > 0$) support the survival of cooperators if they form large enough compact colonies. The visualization of strategy distributions shows that both the number and the average size of these cooperator colonies decrease for these parameters. Consequently, the cooperators become extinct in most simulations. In large systems, however, some C colonies may survive for a long time, allowing the evolution of their payoff parameters to step-by-step help them fend off defectors and thrive.

The numerical results for $L = 2000$ were obtained in a simulation in which a single C colony managed to survive for about 2000 MCSs. During that time, the payoff parameters in the core of this colony evolved as illustrated in Figs. 2 and 3, making the payoff of the cooperators ever higher. Subsequently, the imitators of these cooperators invaded the territory of the defectors and eventually took over the whole system.

Evidently, the same evolutionary process can occur in small systems too, albeit rarely. The probability of whether

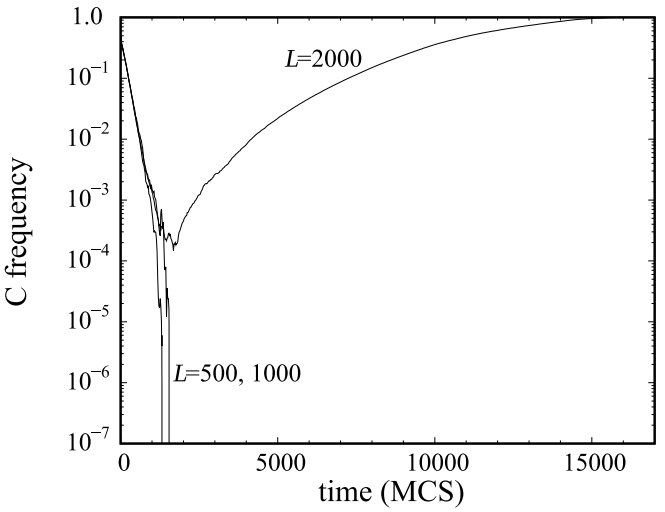

FIG. 5. Time-dependence of C frequency for three system sizes at a high noise level ($K = 1$) and $b = 1$, $c = 0.03$, $r = 0.01$. C and D strategies were randomly distributed in the initial states.

it does depends on the system size (L), the parameters of the dynamical rules (K, δb, δc, and r), as well as the initial spatial distributions of strategies, costs, and benefits. We expect this feature to eventually lead to C dominance in the $L \to \infty$ limit. Establishing the exact, quantitative details of the domain formation and growth processes will require future studies.

It follows from the effective separation of interdomain strategy adoption and intradomain payoff selection that the salient features of how the evolution of the system unfolds should remain unchanged even when the payoff parameters are constrained by a trade-off. The inherently asymmetric structure of the donation game will still determine how payoffs evolve within homogeneous domains: Payoff evolution will remain neutral within homogeneous defector domains, allowing mutation to spread the payoff values within their allowed ranges. Within homogeneous cooperator domains, payoffs will continue to be selected for increasing the reward for mutual cooperation, $b - c$, which may lead to stationary evolutionary states instead of unbounded monotonic parameter drift depending on the specifics of the trade-off. Such an evolutionary process need not transform the initial social dilemma into a harmony game to support cooperation, as illustrated by the conventional model and Fig. 1.

IV. SUMMARY

In the present work, we wished to explore some general features that occur in spatial coevolutionary games in which, in addition to the evolution of the strategies, the parameters of the pair interactions are also allowed to evolve in a way that supports useful innovations. We considered a very simple set of interactions, called donation games, parametrized by just two parameters, costs c and benefits b. Using Monte Carlo simulations, we shed light on the importance of coevolutionary processes promoting innovations that increase the incomes of interacting players in homogeneous strategy distributions that are the preferred absorbing states of strategy imitation. These processes play out very similarly inside of large regions following the same strategy, and thus can significantly affect strategy domain growth.

The curiosity of the donation game lies in the fact that it rewards mutual cooperation with a payoff of $b - c$. Consequently, the adoption of positive innovations in the payoff parameters increases the average benefit and decreases the average cost. In contrast, the incomes of the players when they both defect are independent of the payoff parameters, so both adoption and mutation vary the values of b and c randomly. As a result of this asymmetry, the presence of positive innovations increases the payoffs of cooperative players and eventually suppresses the social dilemma when the cost becomes negative ($c < 0$). The advantage this coevolutionary process offers over the conventional model is clearly reflected in the increased frequency of cooperation.

Just how effectively this specific coevolutionary process can support cooperation relies quite heavily on its interplay with the inherent structural properties of the donation game as a model framework. General symmetric two-strategy two-player games are characterized by four rather than just two payoff parameters, which, after proper rescaling of the baseline and magnitude of the payoffs, can be boiled down to just two universal dilemma strength parameters. The donation game spans only a particular subspace of the parameter plane. Consequently, it can only model interactions that belong to the Harmony (also called Trivial) and Prisoner's Dilemma classes. (For details, see Refs. [12,13,15,20,46,47].) A more general parametrization would give access to the remaining Hawk–Dove (or Chicken or Snowdrift) and Stag-Hunt classes and thus a potentially richer variety of possible evolutionary outcomes.

Nonetheless, its inherent synergy with the donation game does not mean that the coevolutionary mechanism reported in the present paper cannot be relevant in more general settings. Imitation-based strategy adoption typically supports the formation of large domains of absorbing states. Whenever selection affects the coevolutionary variation of payoffs within the different domains differently, it can also influence the spatial competition of the strategies. Our proposed coevolutionary dynamics realizes exactly this by selecting for the increase of only the corresponding payoff entry within each competing homogeneous strategy domain. Consequently, our results also highlight how important the interplay between the underlying interactions and the coevolutionary dynamics can be. The exhaustive quantitative exploration of the resultant phenomena requires further systematic analyses, which are expected to become even more complex when the pair interactions are described by bimatrix games or the evolution follows different dynamical rules. Such analyses would also allow us to better compare and contrast this and a number of recently reported other coevolutionary mechanisms [25–28] that also resolve social dilemma situations by transforming them into harmony games.

DATA AVAILABILITY

There are no publicly available research data or software supporting this paper. Requests for further information or data should be sent to the authors.

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