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Hysteresis modeling

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Abstract

The theoretical modeling in complex physical systems may be aimed at an improved precision of empirical description or a deeper physical understanding of the phenomena. The Preisach-type empirical product model of hysteresis as well as a zero temperature Monte Carlo simulation of the magnetization process of an Ising-like dipolar system are discussed as an illustration of modeling examples. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Relatively well-known principles and laws govern the basic physical properties in a system of magnetic units, thus magnetic phenomena may be modeled and simulated using rather clarified concepts. On the other hand, magnetic systems are complex enough to provide models for the description of conceptually less clarified collective systems. The success of the Ising-model as applied for non-magnetic systems is an obvious example.

As another example, the magnetization curve of a ferromagnet reflects many characteristic features of a whole class of more general non-reversible transition processes (e.g. first-order phase transition, piezoelectric deformation, ferro-electric polarization, gas adsorption and desorption on solid surface, etc.). The irreversible character of the hysteretic phenomena is related to the lag of the hindered response to external actions due to the memory effects of metastable states separated by energy barriers. The scientific knowledge about hysteresis has been developing similar to other physical theories from empirical findings and experimental data collections through phenomenological descriptions of different macroscopic ("top-down") and microscopic ("bottom-up") levels hopefully towards an efficient concise summarizing theory.

Lord Rayleigh's century old parabolic law [1] may be considered as an attempt of 'curve fitting' to known experimental hysteresis curves. Based upon the idea of Rayleigh's law a phenomenological model has been constructed in 1935 by Ferenc Preisach [2] for the calculation of the details of magnetic hysteresis loops in ferromagnetic materials. The Preisach model proved to be surprisingly efficient in describing the most important apparent features of magnetization curves, and has been widely discussed, applied, modified and improved in the literature as seen in a number of useful monographs published on hysteresis modeling during the last decade [3–6]. The Jiles–Atherton model [7] is another example of alternative phenomenological models still at the macroscopic level.

Micromagnetic and ab initio microscopic models concentrate on the intellectual understanding and the physical interpretation of general properties instead of a direct relation to experiments. The classical micromagnetic model of Stoner and Wohlfarth [8] for anisotropic single domain particles or the micromagnetic treatment of reversible bowings and irreversible jumps of domain walls, between pinning centers (e.g. Ref. [9]) are such modeling examples. A new generation of ab initio microscopic models of hysteresis appeared in this decade [10–14] applying zero temperature Monte Carlo simulation of the ferromagnetic Ising model. In such computer experiments the definite jump at the transition field is smeared over by imposed randomness to give realistic looking magnetization transition intervals.

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In this paper, a macroscopic "top-down" and a microscopic "bottom-up" example of hysteresis modeling are to be discussed. The 'product model' [15–18] is derived on the basis of the traditional Preisach model and offers straightforward explanation for non-congruency, saturation and the interconnection of reversible and irreversible magnetization. The magnetic hysteresis was also studied on the microscopic scale with zero temperature Monte Carlo simulation of a three-dimensional Ising system of dipolar particles arranged on a cubic lattice in a spherical volume [14].

2. The product Preisach model

In the Preisach model the magnetic history of a system is determined by a series of consecutive field reversal points $H_0, H_1, \dots H_k, \dots$, and each branch of the magnetization curve connecting such points is calculated as an $E(H_k, H_{k+1})$ Everett integral

$$m(H_{k+1}) - m(H_k) = E(H_k, H_{k+1}) = \int_{H_k}^{H_{k+1}} \mathrm{d}h \int_{H_k}^{h} \mathrm{d}h'\, p(h, h'). \quad (1)$$

The normalized magnetization may change in the $(-1, +1)$ interval and the Preisach function $p(h, h')$ represents the statistical distribution of abstract elementary magnetic particles, called hysterons of given up-switching (h) and down-switching ($h' < h$) fields.

Any complex magnetization history can be followed by summing up the contributions of each unidirectional field change between consecutive reversal points ($H_1, H_2, \dots, H_k, \dots, H_n$):

$$m(H) = \tfrac{1}{2}E(H_1, H_1) + \sum_{k=1}^{n-1} E(H_k, H_{k+1}) + E(H_n, H). \quad (2)$$

The solution of the inverse problem that is the calculation of the Preisach function from the measured data usually involves the mapping of the entire H–m plane [3–6].

The Everett-integrals calculated between the same reversal point limits always give congruent magnetization branches with zero starting slope irrespective of the previous history of field changes and the actual value of m. The congruency property has been considered as an intrinsic feature of the traditional Preisach model [3–6], and it is an obvious consequence of the field-only dependent form: $\mathrm{d}m(h)/\mathrm{d}h = \int_{h_0}^{h} \mathrm{d}h'\, p(h, h')$, of the differential susceptibility derived from Eq. (1).

Some experimental data, however, do not support the validity of the congruency property, and this fact proved to be a weakness of the traditional Preisach model. Of the several attempts for the explanation of non-congruency the "product model" [15–18] is discussed here.

In the product model the congruency property is eliminated by assuming that the susceptibility explicitly depends on both the field and the magnetization. These dependencies are separated into independent multiplying factors:

$$\frac{\mathrm{d}m(h)}{\mathrm{d}h} = R(m)\left\{\beta + \int_{h_0}^{h} \mathrm{d}h'\, q(h, h')\right\}. \quad (3)$$

Experimental data suggest that $R(m)$ should be an even function of m due to symmetry, have a maximum at $m = 0$ and goes to zero when approaching saturation. The assumption, that $R(m)$ is the same for the slope of all branches of a complicated hysteresis curve, is strongly supported by the data of Atherton and Schonbachler [19] measured in a pipeline steel sample.

The two terms of the other factor are both assumed to be independent of m. The term β is the initial susceptibility of the virgin state in the origin and its magnetization contribution changes *reversibly* during the history-dependent integration procedure. The integrand inherits the main properties of the Preisach function and contributes to the *irreversible* magnetization.

The product form of susceptibility suggests, that the magnetization itself should be an indirect function of the applied magnetic field

$$m(h) = G\{\mu(h)\}, \quad \mu = G^{-1}(m). \quad (4)$$

Then, the expression of the differential susceptibility will be given as

$$\frac{\mathrm{d}m(h)}{\mathrm{d}h} = \frac{\mathrm{d}G(\mu)}{\mathrm{d}\mu}\frac{\mathrm{d}\mu(h)}{\mathrm{d}h} \quad (5)$$

and comparing with Eq. (3) we arrive at the result:

$$\frac{\mathrm{d}G(\mu)}{\mathrm{d}\mu} = \frac{\mathrm{d}m}{\mathrm{d}\mu} = R(m), \quad \frac{\mathrm{d}\mu(h)}{\mathrm{d}h} = \beta + \int_{h_0}^{h} \mathrm{d}h'\, q(h, h'). \quad (6)$$

Simple formal integration can provide the form of the magnetization curve branches starting from the demagnetized state ($H_0 = 0$, $m_0 = 0$) through n field reversals up to a general $m(H)$ point:

$$m(H) = G\left\{\beta H + \tfrac{1}{2}F(-H_1, H_1) + \sum_{k=1}^{n-1} F(H_k, H_{k+1}) + F(H_n, H)\right\}, \quad (7)$$

where the $F(H_{k-1}, H_k)$ terms are modified Everett-like integral.

The obtained expression describing the magnetization history is a *transformation* of the Everett integral series — Eq. (2) — for the irreversible part and the βH term represents the reversible part. This combined expression offers a theoretical ground and a practical method for the separation and consequent treatment of the *reversible* and *irreversible* magnetization contributions, which has been mostly an unsolved problem for experts working with measured magnetization curves.

The form of the limiting function $R(m)$ makes sure that *saturation* is a natural *intrinsic property* of the transformation function $G(\mu)$: the argument $\mu(H) = (\beta H + \Sigma F)$ may eventually grow to any large positive or negative values, the magnetization m cannot reach, let alone exceed the saturation value $|m_s| = 1$.

In the special case of uni-axial magnets, one can easily prove [17,18] that the transformation function is identical with the paramagnetic magnetization curve: $m(H) = \tanh(\beta H) = G(\mu(H))$, that is $R(m) = 1 - m^2$. This uni-axial example may suggest a generalization for a more complicated symmetry: in any material the transformation function $G(\beta H)$ and the measured paramagnetic curve would be identical, and in a real ferromagnet the measurement of the *anhysteretic* curve may be proposed as the best possible approximation of the paramagnetic curve. Thus, the measured anhysteretic magnetization curve is identified with the transformation function $G(\mu)$ of the product model and its derivative $R(m) = (\mathrm{d}G/\mathrm{d}H)_n$ (normalized to give $R(0) = 1$) can be used for practical data processing of the measured hysteresis data.

In the product model, the major and minor hysteresis loop branches are totally determined by the H_k and m_k values measured in the kth reversal point, indicating the Markovian character of the consecutive reversal points and loop branches of the magnetization process, not influenced by the details of earlier history.

The limiting function $R(m)$, as deduced from the anhysteretic curve, can be applied then for the derivation of β and $q(h, h')$ from the measured hysteresis data following the traditional method [3–6]. The modified Preisach distribution function $q(h, h')$ of a real material comprises the totality of the information contained in the irreversible part of the hysteresis loops. Its approximation with analytical function forms can provide distribution function parameters, which may be further analyzed and correlated to the microscopic properties of the material.

3. The dipolar Ising model

Using the techniques of statistical physics we have studied an Ising system of point-like magnetic dipoles by Monte Carlo simulation and it was shown that this model exhibits magnetic hysteresis in periodically varying external magnetic field [14].

A three-dimensional Ising model is considered with spins located on the points of a simple cubic lattice ($\boldsymbol{r} = (x, y, z)$; x, y, and z are integers) within a sphere of radius $\boldsymbol{R}$ and interacting by the nearest-neighbor exchange as well as the long-range dipole–dipole couplings. With dimensionless quantities the Hamiltonian is given by

$$H = -J\sum_{(r,r')}\sigma(\boldsymbol{r})\sigma(\boldsymbol{r}') + \frac{1}{2}\sum_{\substack{r,r' \\ r\neq r'}}\frac{|\boldsymbol{r}-\boldsymbol{r}'|^2 - 3|z-z'|^2}{|\boldsymbol{r}-\boldsymbol{r}'|^5}\sigma(\boldsymbol{r})\sigma(\boldsymbol{r}') - h\sum_{r}\sigma(\boldsymbol{r}), \tag{8}$$

where $\sigma(r) = 1$ and -1 for up and down spins. The energy terms represent the exchange and the dipole–dipole interaction between the spins having only z component, and the energy of a spin in the external magnetic field. The dipole–dipole coupling is ferromagnetic along the z-axis and antiferromagnetic in the x–y plane.

The dipolar energy ($h = 0$ and $J = 0$) was determined by Luttinger and Tisza [20,21] for ordered periodic spin configurations in an infinite cubic lattice. In the energetically favored states the up (or down) spins form vertical columns as expected. The lowest energy structure is a twofold degenerated chess-board-like antiferromagnetic (CAF) arrangement of ferromagnetic columns. The next lowest energy structure is a fourfold degenerated, layered antiferromagnetic (LAF) spin configuration of ferromagnetic planes. The slow cooling Monte Carlo technique has confirmed the formation of a structure equivalent to the chess-board-like twofold degenerated ground state. For both (CAF and LAF) configurations the finite size corrections are found proportional to $1/R$.

A series of zero temperature Monte Carlo simulations has been performed to study the hysteresis phenomena starting from a random spin configuration. In an elementary process a randomly chosen spin is flipped if the direction of the sum of the calculated dipolar and external fields is opposite to the spin direction. This process is repeated until all the spins point to the direction of the local magnetic field. Then the external magnetic field (h) is increased (decreased) by Δh and the relaxation process is started again proceeding toward a new local energy minimum. The external field is varied periodically with an over-saturating amplitude of 10. During this procedure the magnetization of the sphere exhibits irreversible magnetic hysteresis.

By varying the magnetic field and, if applicable, reversing a randomly chosen spin of the actual configuration, the local field will be modified in all sites of the system. Due to the dipole–dipole interaction the neighboring spins inside the same column are favored to be reversed too. This effect drives an avalanche of spin flips in the given column leading predominantly to such configurations where complete spin columns have been reversed. Avalanches were observed in experiments by Barkhausen [22] and also in the random field Ising models [23].

The initially saturated state ($h = 10$, $m = 1$) remains unchanged with decreasing magnetic field until $h = 2.253$,

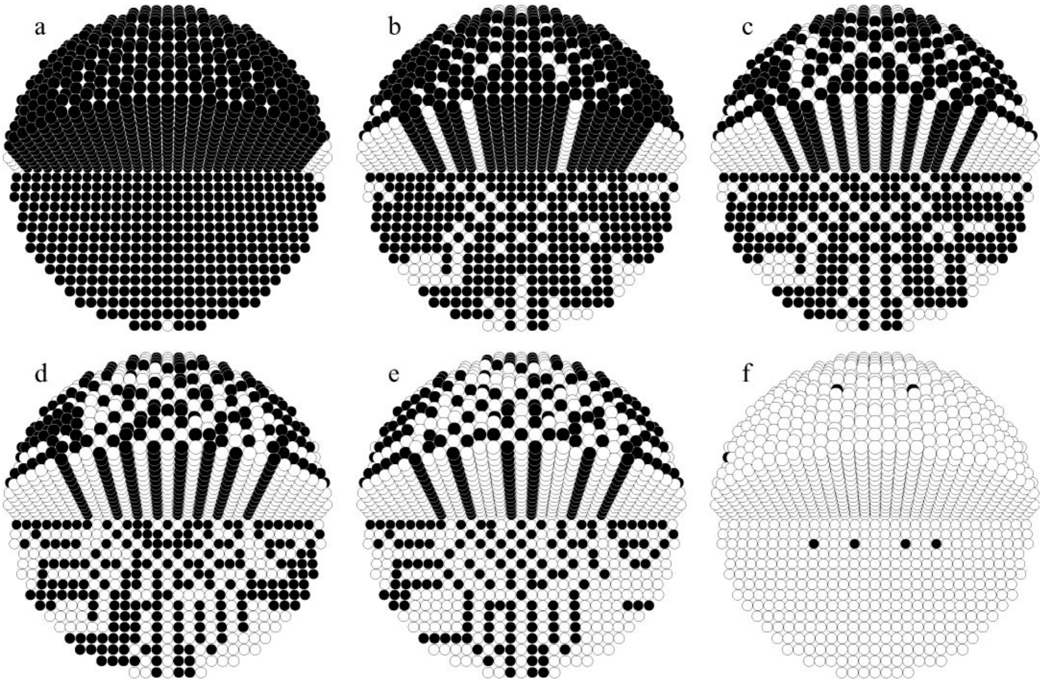

Fig. 1. The configuration of up (black) and down (white spheres) spins during a field change from 10 down to 2 (a); 1.7 (b); 0 (c); -1 (d); -4 (e) and -6 (f).

a value depending slightly on R. Then some columns switch to opposite, starting from typical symmetric positions along the equatorial line of the sphere. A typical state with remnant magnetization is formed at $h = 0$, and the magnetization vanishes at about $h \cong -1$. Here, the overall magnetic structure consists of domains with CAF structure in the central region and those of LAF structure near the equatorial line. These domains remain recognizable in a wide range of the magnetization process even for $h = -2$. Further decrease of h will destroy these ordered regions leaving the spins unchanged only in a few columns positioned randomly. Finally, all the spins point to downward if the magnetic field becomes less than -6.2. The process can be seen in the Fig. 1.

4. Conclusion

Two examples of hysteresis modeling have been presented.

The product model is an *output-dependent* modification of the traditional Preisach model in which the *congruency* property is removed, the *saturation* is an intrinsic natural property of the magnetization curves due to the applied mathematical transformation, and the *reversible and irreversible* contributions of the magnetization are composed and treated together in the argument of an indirect function. The overall behavior of the hysteresis curve is related to the *anhysteretic* magnetization, which represents the theoretical paramagnetic process. The model parameters characterizing the microscopic properties of the material can be determined from the experimental data by the well-known evaluation procedure.

In the other example a zero temperature Monte Carlo simulation of the magnetization process exhibits hysteresis and avalanches. The process of avalanche is constrained in a few columns. As a result, the number of possible stationary metastable states is decreased drastically in comparison with the total number of configurations, which is a characteristic feature of the hysteretic behavior.

The presented models may be good candidates for exploring the relationship between the phenomenological approaches and the microscopic descriptions of hysteresis.

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