Improving power-grid systems via topological changes or how self-organized criticality can help power grids

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Strategic research area project of ELKH





Energiatudományi Kutatóközpont



Nemzeti Kutatási, Fejlesztési És Innovációs Hivatal

Dynamic Days 2024

Géza Ódor, István Papp, Kristóf Benedek, and Bálint Hartmann Phys. Rev. Research 6, (2024) 013194, Smart E. and Grids 2024

Large-scale blackouts in the world and their

consequences

No.	Country	Year	Load loss (GW)	Economic loss	People affected (*Million)	Duration (hours)	Reference
1	Iran	2003	~7	Not available	22	8	[10, 13]
2	USA, Canada	2003	61.8	\$ 6.4 billion	50	16–72 (USA), up to 192 (Canada)	[10-12]
3	Italy	2003	24	Over €120 million	~ 56	Up to ~18	[10, 12]
4	Russia	2005	~3.5	\$ 1-2 billion	4	~4	[46, 50]
5	Western Europe	2006	~14	Not available	15	~2	[12]
6	USA and Mexico	2011	4.3	Up to \$118 million	Over 5	~11	[50]
7	India	2012	~48	Not available	670	2-8	[13, 51]
8	Turkey	2015	32.2	Not available	70	More than 7	[13]

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Renewable energy adds more instability due to the large fluctuations of the power sources, like wind farms:

Anvari et al 2018 Complexity and Synergetics.





Figure 4. Probability distribution function of energy unserved for North American blackouts 1993-1998.

B. Carreras et al, Proceedings of Hawaii International Conference on System Sciences, Jan. 4-7, 2000, Maui, Hawaii. 2000 IEEE





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Extreme events occur more frequently than by an independent variable ensemble



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Competition of supply and demand



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TABLE I. Observed and simulated power law exponents in the noncumulative pdf of blackout size. The power law exponent is often calculated by subtracting one from an estimate of the slope of a log-log plot of a complementary cumulative probability distribution.

Source	Exponent	Quantity
North America data (Ref. 6)	-1.3 to -2.0	Various
North America data (Refs. 19 and 20)	-2.0	Power
Sweden data (Ref. 21)	-1.6	Energy
Norway data (Ref. 22)	-1.7	Power
New Zealand data (Ref. 23)	-1.6	Energy
China data (Ref. 24)	-1.8	Energy
	-1.9	Power
OPA model on tree-like 382-node (Ref. 8)	-1.6	Power
Hidden failure model on WSCC 179-node (Ref. 9)	-1.6	Power
Manchester model on 1000-node (Ref. 10)	-1.5	Energy
CASCADE model (Ref. 11)	-1.4	No. of failures
Branching process model (Ref. 12)	-1.5	No. of failures

Dobson et al Chaos 17 (2007) 026103

The EU 2016 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data





FIG. 9. All nodes of the European power-grid 2016 data sep a rated into 12 communities, taking into account admittance using a giant component of 13 478 nodes connected by 18 39 links, maintaining the modularity score close to the maximum $Q \approx 0.795$.



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FIG. 9. All nodes of the European power-grid 2016 data sep a rated into 12 communities, taking into account admittance using a giant component of 13 478 nodes connected by 18 395 links, maintaining the modularity score close to the maximum $Q\approx 0.795.$



Modular HV network, with graph dimension d = 2.6(1), but $d_s < 2$

The US 2016 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data





FIG. 12. All nodes of the USA power-grid 2016 gauge grant component, separated into 12 communities, taking into account the admittances and 14 990 nodes connected by 20 880 edges, maintaining the modularity score $Q \approx 0.859$ with resolution $\Gamma = 1 \times 10^{-4}$.

Adjacency Matrix

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FIG. 12. All nodes of the USA power-grid 2016 uata gram component, separated into 12 communities, taking into account the admittances and 14 990 nodes connected by 20 880 edges, maintaining the modularity score $Q \approx 0.859$ with resolution $\Gamma = 1 \times 10^{-4}$.

Adjacency Matrix

Modular HV network, with with graph dimension d = 2.4(1)

The EU 2022 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data



FIG. 11. All nodes of the European power-grid 2022 data giant component, separated into 10 communities, taking into account the admittances and 7411 nodes connected by 10912 edges without smaller voltage level edges, maintaining the modularity score $Q \approx 0.854$.



Adjacency Matrix

The EU 2022 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data







Adjacency Matrix

Modular HV network, with graph dimension: d = 1.8(2)

Incomplete network

Summary of network invariants

G = (V, E) N nodes, E edges Graph: $\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_{i_i}$ Average degree $P(k_i > K) = C \cdot e^{-\frac{k_i}{\gamma_c}}$ Cumulative degree distribution $L = \frac{1}{N(N-1)} \sum_{i \neq i} d(i,j), \qquad L_r = \frac{\ln N - 0,5772}{\ln \langle k \rangle} + 1/2$ Shortest path-length $C = \frac{1}{N} \sum_{i} \frac{2n_i}{k_i} \left(\frac{k_i}{k_i} - 1 \right)$ Clustering coefficient $\sigma = \frac{C/C_r}{L/L_r} \quad \text{Modularity} \quad Q = \frac{1}{N\langle k \rangle} \sum_{ii} \left(A_{ij} - \Gamma \frac{k_i k_j}{N\langle k \rangle} \right) \delta(g_i, g_j).$ Small world coefficient Network *E N* $\langle k \rangle$ γ_c *Q* Community # *L L*_r *C C*_r σ 92.702 1,504 0,924 28 49,50 9,396 0,099 0,000203 EU16 18393 13478 2.729 2,779 1,640 0,849 12 46,83 8,653 0,098 EU22 48,420 10298 7411 0,000375 14990 2,786 1,548 0,927 22 47,50 9,321 0,102 0,000186 107,785 USA16 20880

Similar invariants, small world networks, but d < 3

The synchronization model

- Blackouts can be modeled by desynchronization of AC power grids
- Power transmission: a mismatch "Δθ" in the phases between "G" and "M" ⇒ the Kuramoto model with inertia ¹:

$$P_{\text{source}} = P_{\text{acc.kinetic}} + P_{\text{diss.}} + P_{\text{transmitted}}$$

$$= \frac{1}{2} I \frac{d}{dt} \dot{\theta}_{1}^{2} + P_{\text{diss.}} - P^{\text{MAX}} \sin(\Delta \theta)$$

$$\Rightarrow \ddot{\theta}_{1} = P - \alpha \dot{\theta}_{1} + P^{\text{MAX}} \sin(\Delta \theta). \quad (1) \quad \text{M} _{\theta_{2}} \quad \text{M} _{\theta_{3}} \quad \text{M} _{\theta_{4}}$$

For a network of N oscillators:

$$\dot{\theta}_i(t) = \omega_i(t)$$

$$\dot{\omega}_i(t) = \omega_i(0) - \alpha \dot{\theta}_i(t) + K \sum_{j=1}^N A_{ij} \sin \left[\theta_j(t) - \theta_i(t)\right] .$$
(2)

 α : damping factor; *K*: global coupling; $\omega_i(0) \sim N(0,1)$

¹ G. Filatrella et al., Eur. Phys. J. B, 61, 485-491 (2008).

Methods and measured quantities

- For large N, solved Eqs. (2) by numeric solvers: 4th-order Runge-Kutta, Bulirsch-Stoer
- For large K: adaptive Bulirsch-Stoer
- GPU code (kuramotoGPU) by utilizing VexCL's vector capability.

Measured quantities

Phase order parameter

$$z(t_k) = \frac{1}{N} \left| \sum_{j} \exp\left[i\theta_j(t_k) \right] \right|$$
$$R(t_k) = \langle r(t_k) \rangle.$$
(3)

2 Frequency variance: $\Omega(t_k) = \langle \operatorname{var}(\omega_i(t_k)) \rangle$.

¹ H. Hong et al., Phys. Rev. E, 72, 036217 (2005).

² G. Ódor and B. Hartmann. Phys. Rev. E, 98 022305 (2018).

Cascade simulations by line-cuts

After thermalization, randomly remove a link w.r.t. the overload condition:



 $|\sin(\theta_j - \theta_i)| > T \Rightarrow A_{ij} := 0.$

EU network K = 80

O Stronger damping effect only slows down R, but leads to a smaller Ω .

Por certain T, R may even increase: islanding effects?

Cascade failure statistics



 The distribution of the total line failures N_f follows non-universal power laws in the vicinity of (K_c, T_c)

 $p(N_f) \sim N_f^{-\tau}$.

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FIG. 9. Fluctuations of $R(t \to \infty)$ in case of the U.S.-HV power grid at the end of the thermalization process. Both for the normal $\alpha = 0.4$ and the $\alpha = 3$ dissipation cases. Partial synchronization transition occurs at $K_c \simeq 22(2)$. The inset shows the decay of $\Omega(t)$ starting from disordered states at $\alpha = 3$ and for different global couplings, as shown by the legends. The curves are multiplied by a factor $t^{1.5}$ in order to see the scaling at $K_c = 20$.



FIG. 9. Fluctuations of $R(t \to \infty)$ in case of the U.S.-HV powe grid at the end of the thermalization process. Both for the norma $\alpha = 0.4$ and the $\alpha = 3$ dissipation cases. Partial synchronization transition occurs at $K_c \simeq 22(2)$. The inset shows the decay of $\Omega(t)$ starting from disordered states at $\alpha = 3$ and for different globa couplings, as shown by the legends. The curves are multiplied by a factor $t^{1.5}$ in order to see the scaling at $K_c = 20$.

FIG. 16. Relative change of the steady-state Kuramoto order parameter at $\alpha = 0.4$ the consequence of the cascade in the Europe-HV grid. The R(T = 1) values are set to be the reference points and R(T)/R(T = 1) is plotted. The gray dashed line marks the baseline for the emergence of islanding effects. The inset shows $\sigma[R(T, K)]$.



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Enhanced stability at $\sigma(R)$ peaks, near the synchronization transition !

Comparison of frequency results with measurements of Hungarian HV power-grid

50100

All locations

q-Gaussian

Gaussian

50000

f[mHz]

50050

BEKO

DETK

GYOR1



Frequency data in Hungary at 10/23 2022 Fits well with q-Gaussian $(q \sim 1.1)$ (SEGAN)

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Kuramoto solution on a 387 node HV with Real pararameters, in/out powers, line admittances, inertias ... etc



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Kuramoto solution on a 387 node HV with Real pararameters, in/out powers, line admittances, inertias ... etc Fits well with q-Gaussian *(q~1.6)*





$$r_i(t) = \frac{1}{N_{i.\text{neigh}}} \left| \sum_{j}^{N_{i.\text{neigh}}} A_{ij} e^{i\theta_j(t)} \right|$$

Local frequency Synchronization during a blackout cascade, simulated by kuramotoGPU. *Chimera states: Deng and Ódor Chaos 2024*



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Improve by adding community bridges



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Or adding bypasses at weak local synchronization nodes



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Or adding bypasses at weak local synchronization nodes **Dynamic**



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Improve by adding community bridges Static



FIG. 5. Comparison of dynamical simulation results of R at the end of the thermalization for the original, randomly extended, bridged and bypassed networks, using $\alpha = 0.4$.



Or adding bypasses at weak local synchronization nodes **Dynamic**

Braess Paradox and the SOC

Expansion plans in real power grids cause non-local overloads of the grid.



Schafer et al Nature Communications 13, 5396 (2022) 5396
Braess Paradox and the SOC



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Efficient GPU synchronization simulations to describe blackouts on large AC networks

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Non-local power-law cascade distributions, Dragon Kings, Island effects

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Phys. Rev. Res. 6 (2024) 013194 SEGAN 2024 in press

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Recent, related publications:

Géza Ódor, Shengfeng Deng, ynchronization Transition of the Second-Order Kuramoto Model on Lattices Entropy 25 (2023), 164

Géza Ódor, Shengfeng Deng, Balint Hartmann and Jeffrey Kelling Synchronization dynamics on power grids in Europe and the United States, **Physical Review E 106 (2022) 034311.**

Géza Ódor and Bálint Hartmann Power-Law Distributions of Dynamic Cascade Failures in Power-Grid Models, **Entropy 22 (2020) 666**

Géza Ódor and Bálint Hartmann, Heterogeneity effects in power grid network models **Phys. Rev. E 98 (2018) 022305**

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Outage durations with PL tails (digression)

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Electrical outages \neq Blackout cascades, still they show PL duration



FIG. 2. Probability distributions (black dots) of generation outages measured in terms of the unavailable duration. For the ENTSO-E data, we show the generation outage data for the control areas "DE_AMPRION", "GB", and "FR", as well as the generation and production outage data from all control areas. The fitted power laws and their corresponding x_{\min} values are marked by solid red lines and vertical black lines, respectively.

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Following power-spectral analysis we proposed SOC and HOT models to understand See our recent paper : **PRX Energy 2 (2023).033007**

Summary of community results

Community	Size (EU22)	$\langle k \rangle$ (EU22)	Size (EU16)	$\langle k \rangle$ (EU16)	Size (US16)	$\langle k \rangle$ (US16)
1	924	2.72	4285	2.83	3511	2.79
2	479	2.70	2526	2.66	2829	2.98
3	2016	2.84	1527	2.67	1640	2.72
4	698	3.06	1461	2.72	1484	2.69
5	595	2.94	1455	2.69	1396	2.93
6	1059	2.66	966	2.77	1165	2.58
7	1237	2.68	638	2.57	768	2.97
8	16	2.81	289	2.06	710	2.57
9	332	2.18	277	2.99	673	2.70
10	55	2.74	26	3.07	390	2.84
11	-	-	22	3.31	230	2.43
12	-	-	6	2.66	194	2.69

TABLE I. Community sizes and average degrees for different data-sets, for the resolution $\Gamma = 10^{-4}$. We refer to sizes here as number of nodes in the respecting community. These structures correspond to the maps plotted on Figs.9, 11, 12.

Louvain algorithm used with resolution Parameter Γ

$$Q = \frac{1}{N\langle k \rangle} \sum_{ij} \left(A_{ij} - \Gamma \frac{k_i k_j}{N\langle k \rangle} \right) \delta(g_i, g_j),$$

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FIG. 8. Community size distributions at different Γ resolution parameters for different networks shown in the legend.

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FIG. 14. Probability distributions of the inverse of cable lengths of the European and North-American SciGRID networks. Left inset: the same data plotted on the $-\ln(p)$ scale to compare with the stretched exponential assumption, that would correspond to a straight line tail, Right inset: probability distributions of the line lengths in meters.





FIG. 14. Probability distributions of the inverse of cable lengths of the European and North-American SciGRID networks. Left inset: the same data plotted on the $-\ln(p)$ scale to compare with the stretched exponential assumption, that would correspond to a straight line tail, Right inset: probability distributions of the line lengths in meters.

$$R_{ij} = \left(\frac{U_c}{U_{ij}}\right)^2 \cdot L_{ij} \cdot R_{c_k} \qquad P_{ij} = P_{c_k}$$
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TABLE II. Characteristic values of relevant physical quantities in the modeled European power grids.

Voltage [kV]	$R_c [\Omega/km]$	$X_c [\Omega/km]$	$C_c [nF/km]$	P _c [MW]
120	0.0293	0.1964	9.4	170
220	0.0293	0.2085	9.0	360
380/400	0.0286	0.3384	10.8	1300

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220	0.0293	0.2085	9.0	360
380/400	0.0286	0.3384	10.8	1300



$$R_{ij} = \left(\frac{U_c}{U_{ij}}\right)^2 \cdot L_{ij} \cdot R_{c_k} \qquad P_{ij} = P_{c_k},$$
$$X_{ij} = \left(\frac{U_c}{U_{ij}}\right)^2 \cdot L_{ij} \cdot X_{c_k} \qquad W_{ij} = \frac{P_{ij}}{X_{ij}} / \left\langle\frac{P}{X}\right\rangle$$



FIG. 14. Probability distributions of the inverse of cable lengths of the European and North-American SciGRID networks. Left inset: the same data plotted on the $-\ln(p)$ scale to compare with the stretched exponential assumption, that would correspond to a straight line tail, Right inset: probability distributions of the line lengths in meters.

TABLE II. Characteristic values of relevant physical quantities in the modeled European power grids.

Voltage [kV]	$R_c [\Omega/km]$	$X_c [\Omega/km]$	$C_c [nF/km]$	P_{c} [MW]
120	0.0293	0.1964	9.4	170
220	0.0293	0.2085	9.0	360
380/400	0.0286	0.3384	10.8	1300



PL exponents ~ 2, Universal ?

Load and generator power distributions



FIG. 5. Distribution of nodal generations and loads of the ENTSO-E 2016 database. Power-law fits were applied to the [20...300] MW range in the inset figure. The exponents of the fits are: y = 1.16(5) both for generation and load curves, respectively. The load data shows an earlier size cutoff, which is an important characteristic of traditional power systems, where energy is produced in a centralized manner by large power plants to increase efficiency, and energy is consumed in a distributed manner. The main figure shows the same data, with stretched exponential fits, according to Eq. (7) in the range [10...1000] MW

$$p(k) \propto \exp\left(-(k/B)^{\beta}\right)$$

Load and generator power distributions



FIG. 5. Distribution of nodal generations and loads of the ENTSO-E 2016 database. Power-law fits were applied to the [20...300] MW range in the inset figure. The exponents of the fits are: y = 1.16(5) both for generation and load curves, respectively. The load data shows an earlier size cutoff, which is an important characteristic of traditional power systems, where energy is produced in a centralized manner by large power plants to increase efficiency, and energy is consumed in a distributed manner. The main figure shows the same data, with stretched exponential fits, according to Eq. (7) in the range [10...1000] MW



FIG. 6. Distribution of nodal generations and loads of the 2021 US [69] database. Inset: different power-law fits were applied to the [5...200] MW for generators and [50...200] MW for loads. The load data shows an earlier size cutoff as for the European case. The main figure shows the same data with stretched exponential fit according to Eq. (7) in the range [20...500] MW.

 $p(k) \propto \exp\left(-(k/B)^{\beta}\right)$
Load and generator power distributions



FIG. 5. Distribution of nodal generations and loads of the ENTSO-E 2016 database. Power-law fits were applied to the [20...300] MW range in the inset figure. The exponents of the fits are: y = 1.16(5) both for generation and load curves, respectively. The load data shows an earlier size cutoff, which is an important characteristic of traditional power systems, where energy is produced in a centralized manner by large power plants to increase efficiency, and energy is consumed in a distributed manner. The main figure shows the same data, with stretched exponential fits, according to Eq. (7) in the range [10...1000] MW



FIG. 6. Distribution of nodal generations and loads of the 2021 US [69] database. Inset: different power-law fits were applied to the [5...200] MW for generators and [50...200] MW for loads. The load data shows an earlier size cutoff as for the European case. The main figure shows the same data with stretched exponential fit according to Eq. (7) in the range [20...500] MW.

 $p(k) \propto \exp\left(-(k/B)^{\beta}\right) \qquad \beta \sim 0.25$ Universal ?