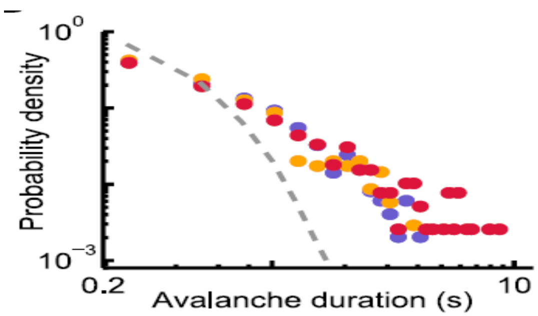
Power-law tailed weight distributions in connectome graphs

Géza Ódor, István Papp
EK-MFA Complex Systems Department, Budapest
Michael T. Gastner, Singapore Institute of Technology
Gustavo Deco, University Pompeu Farba, Barcelona, Spain

Power-law tailed weight distributions in connectome graphs

Géza Ódor, István Papp
EK-MFA Complex Systems Department, Budapest
Michael T. Gastner, Singapore Institute of Technology
Gustavo Deco, University Pompeu Farba, Barcelona, Spain

Theoretical research and experiments suggest that the brain operates at or near a **critical state** between sustained activity and an inactive phase, exhibiting optimal computational properties (see: *Beggs & Plenz J. Neurosci. 2003; Chialvo Nat. Phys. 2010; Haimovici et al. PRL 2013*)

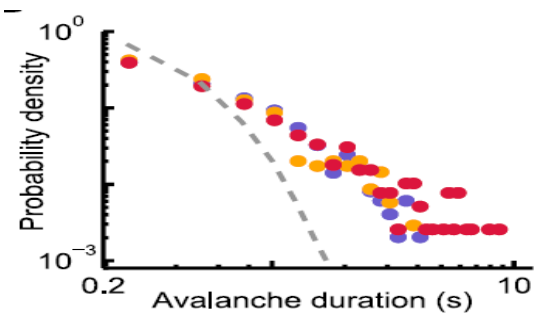


Power-law tailed weight distributions in connectome graphs

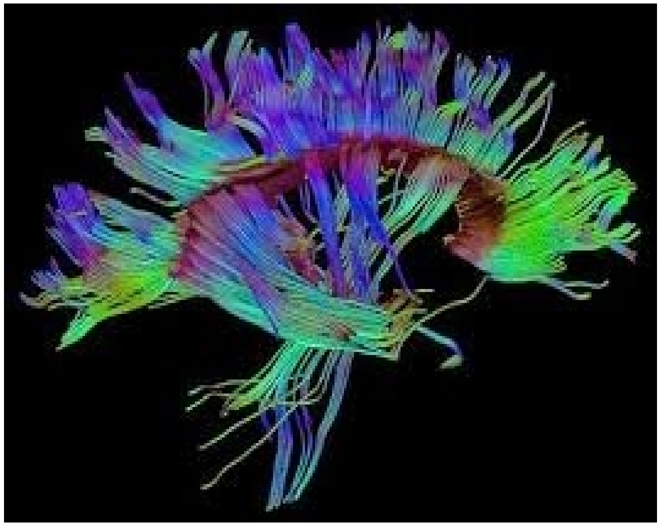
Géza Ódor, István Papp
EK-MFA Complex Systems Department, Budapest
Michael T. Gastner, Singapore Institute of Technology
Gustavo Deco, University Pompeu Farba, Barcelona, Spain

Theoretical research and experiments suggest that the brain operates at or near a **critical state** between sustained activity and an inactive phase, exhibiting optimal computational properties (see: *Beggs & Plenz J. Neurosci. 2003; Chialvo Nat. Phys. 2010; Haimovici et al. PRL 2013*)

Criticality → PL distributed spatial and temporal correlation lengths



Open Connectome (OC) Large Human graphs



Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

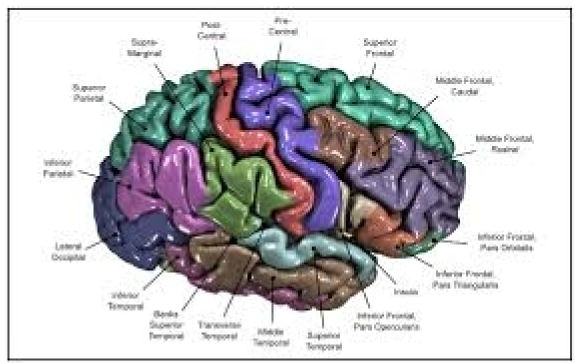
Hierarchical modular graphs

Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

Hierarchical modular graphs

Top level: *70* brain region (Desikan atlas)



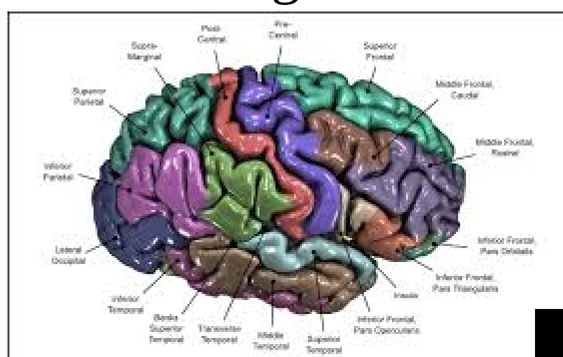
Open Connectome (OC) Large Human graphs

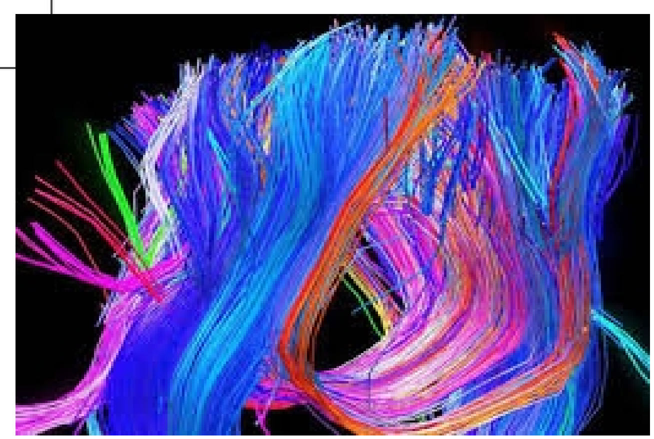
Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

Hierarchical modular graphs

Top level: *70* brain region (Desikan atlas)

Lower levels: Deterministic tractography:
Fiber Assignment by Continuous Tracking (FACT) algorithm





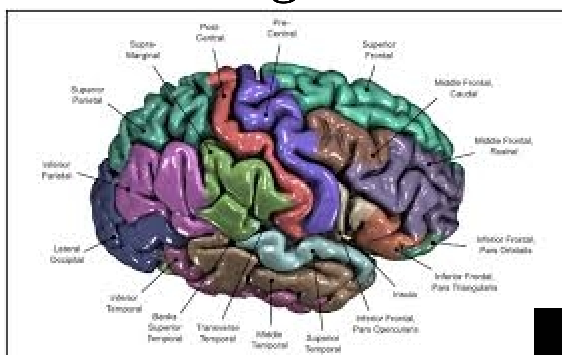
Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10^5 –10^6 nodes

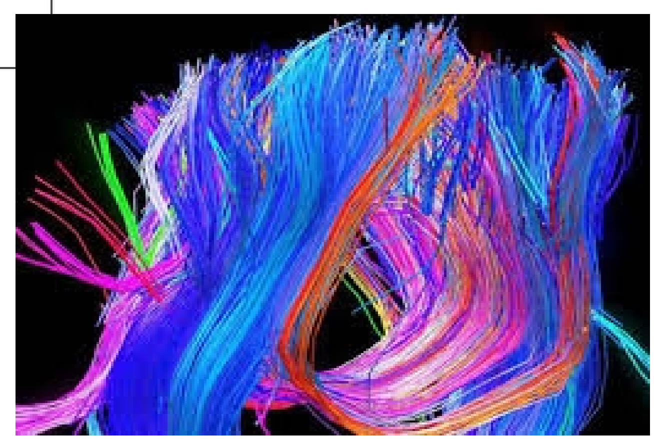
Hierarchical modular graphs

Top level: *70* brain region (Desikan atlas)

Lower levels: Deterministic tractography:
Fiber Assignment by Continuous Tracking (FACT) algorithm



Map : voxel → vertex (~ *10^7*)



Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

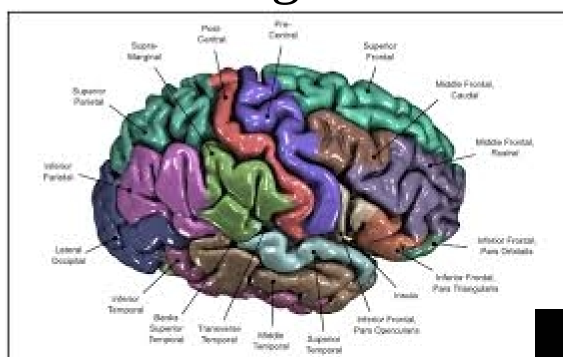
Hierarchical modular graphs

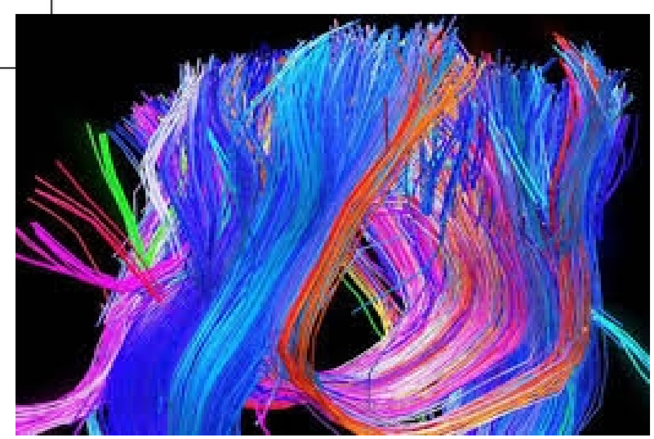
Top level: *70* brain region (Desikan atlas)

Lower levels: Deterministic tractography:
Fiber Assignment by Continuous Tracking (FACT) algorithm

Map : voxel → vertex (~ *10* ^{7})

fiber → edge (~ *10* ^{10})





Open Connectome (OC) Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} ***–10*** ^{6} nodes

Hierarchical modular graphs

Top level: *70* brain region (Desikan atlas)

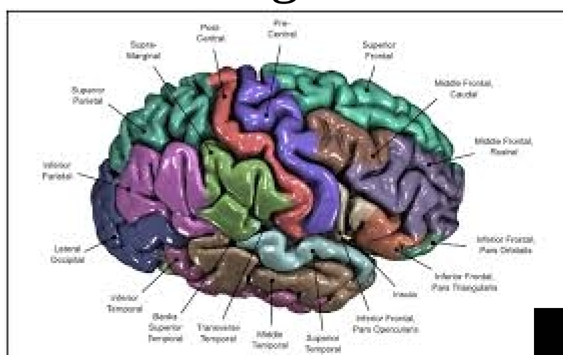
Lower levels: Deterministic tractography:
Fiber Assignment by Continuous Tracking (FACT) algorithm

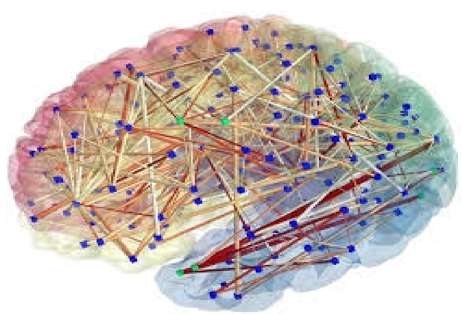
Map : voxel → vertex (~ 10^7)

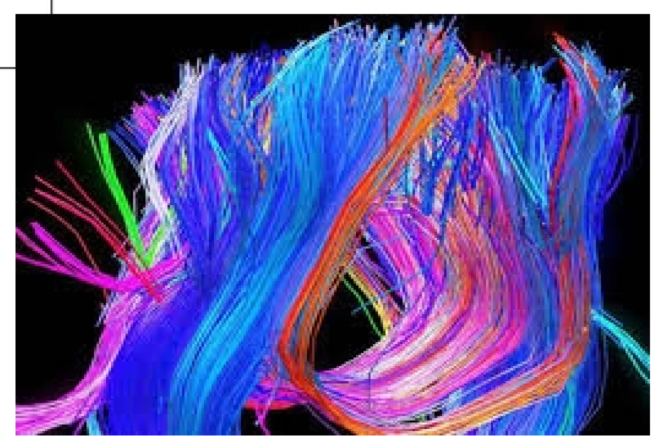
fiber → edge (~ 10^{10})

+ noise reduction → graph

undirected, weighted







The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The degree distribution of ***10*** large human connectomes obtained by DMRI DTR algorithm has been analyzed by Max likelihood + Akaike Criterion, for ***6*** models:

The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The degree distribution of ***10*** large human connectomes obtained by DMRI DTR algorithm has been analyzed by Max likelihood + Akaike Criterion, for ***6*** models:

$$\mathcal{L}(\mathbf{v}) = \prod_{i=k_{\min}}^{k_{\max}} [\Pr(k_i, \mathbf{v})]^{n_i},$$

The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The degree distribution of ***10*** large human connectomes obtained by DMRI DTR algorithm has been analyzed by Max likelihood + Akaike Criterion, for ***6*** models:

$$\mathcal{L}(\mathbf{v}) = \prod_{i=k_{\min}}^{k_{\max}} [\Pr(k_i, \mathbf{v})]^{n_i},$$

$$\mathrm{AIC}_c = -2\ \ln(\mathcal{L}(\hat{\mathbf{v}})) + 2K + \frac{2K(K+1)}{N-K-1}$$

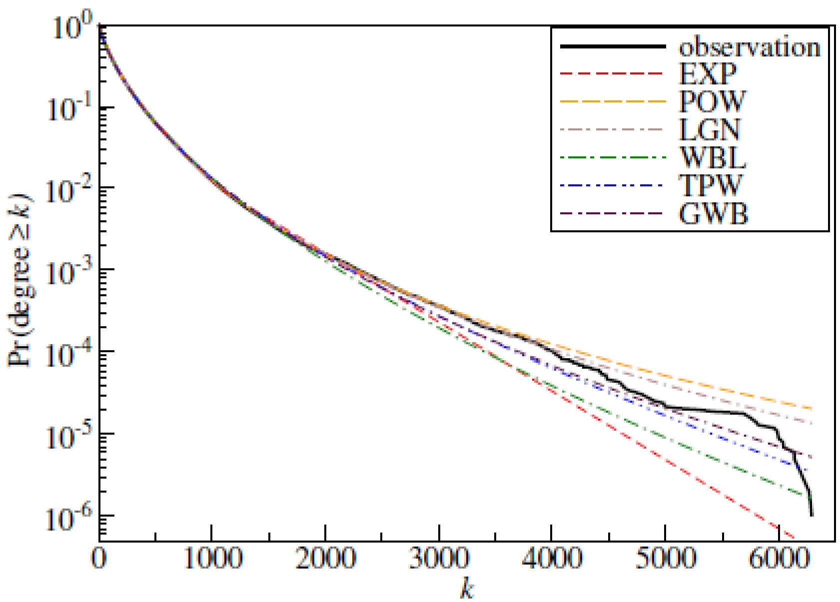
Model	$F(k)$
exponential (EXP)	$e^{-\alpha k}$
power law (POW)	$\alpha^{\beta}(k+\alpha)^{-\beta}$
log-normal (LGN)	$\frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{\ln k - \alpha}{\sqrt{2}\beta}\right)$
Weibull (WBL)	$\exp(-\alpha k^{\beta})$
truncated power law (TPW)	$\alpha^{\beta}(k+\alpha)^{-\beta}e^{-\gamma k}$
generalized Weibull (GWB)	$\exp[\alpha(\gamma^{\beta}-(k+\gamma)^{\beta})]$

The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The degree distribution of ***10*** large human connectomes obtained by DMRI DTR algorithm has been analyzed by Max likelihood + Akaike Criterion, for ***6*** models:



$$\mathcal{L}(\mathbf{v}) = \prod_{i=k_{\min}}^{k_{\max}} [\Pr(k_i, \mathbf{v})]^{n_i},$$

$$\mathrm{AIC}_c = -2\ln(\mathcal{L}(\hat{\mathbf{v}})) + 2K + \frac{2K(K+1)}{N-K-1}$$

Model	$F(k)$
exponential (EXP)	$e^{-\alpha k}$
power law (POW)	$\alpha^{\beta}(k+\alpha)^{-\beta}$
log-normal (LGN)	$\frac{1}{2}-\frac{1}{2}\,\mathrm{erf}\left(\frac{\ln k-\alpha}{\sqrt{2}\beta}\right)$
Weibull (WBL)	$\exp(-\alpha k^{\beta})$
truncated power law (TPW)	$\alpha^{\beta}(k+\alpha)^{-\beta}e^{-\gamma k}$
generalized Weibull (GWB)	$\exp[\alpha(\gamma^{\beta}-(k+\gamma)^{\beta})]$

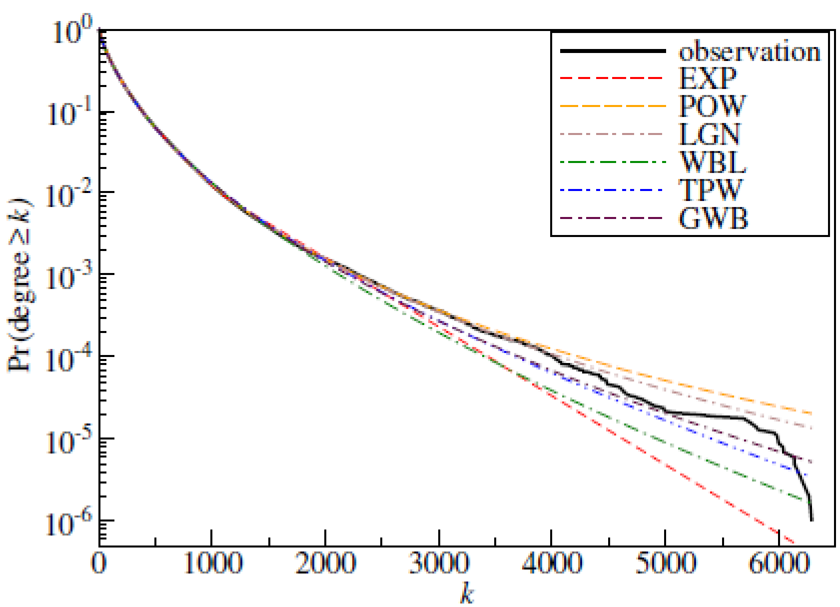
Figure 2. The maximum-likelihood distributions from each model family for matching the degree distribution of network KKI-18. In this example the generalized Weibull distribution is the best compromise in the right tail (see Table 2).

The topology of large Open Connectome networks for the human brain

Michael T. Gastner[1,2] & Géza Ódor[2]

SCIENTIFIC REPORTS | 6:27249 | DOI: 10.1038/srep27249

The degree distribution of ***10*** large human connectomes obtained by DMRI DTR algorithm has been analyzed by Max likelihood + Akaike Criterion, for ***6*** models:



$$\mathcal{L}(\mathbf{v}) = \prod_{i=k_{\min}}^{k_{\max}} [\Pr(k_i, \mathbf{v})]^{n_i},$$

$$\mathrm{AIC}_c = -2\ln(\mathcal{L}(\hat{\mathbf{v}})) + 2K + \frac{2K(K+1)}{N-K-1}$$

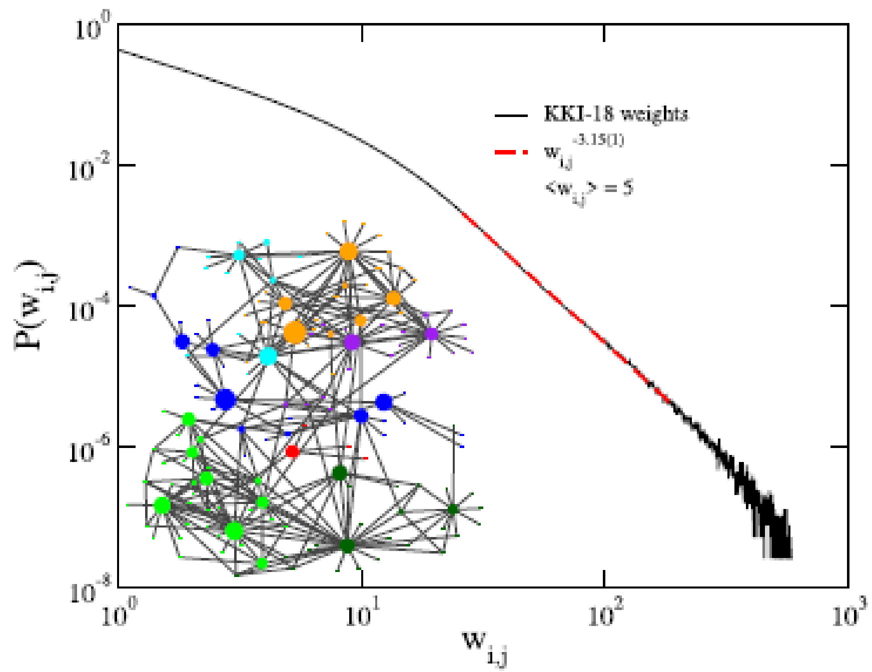
Model	$F(k)$
exponential (EXP)	$e^{-\alpha k}$
power law (POW)	$\alpha^{\beta}(k+\alpha)^{-\beta}$
log-normal (LGN)	$\frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{\ln k - \alpha}{\sqrt{2}\beta}\right)$
Weibull (WBL)	$\exp(-\alpha k^{\beta})$
truncated power law (TPW)	$\alpha^{\beta}(k+\alpha)^{-\beta}e^{-\gamma k}$
generalized Weibull (GWB)	$\exp[\alpha(\gamma^{\beta} - (k+\gamma)^{\beta})]$

Figure 2. The maximum-likelihood distributions from each model family for matching the degree distribution of network KKI-18. In this example the generalized Weibull distribution is the best compromise in the right tail (see Table 2).

The generalized Weibull (stretched exponential) fits the best them

Weight (number of edges) distributions for large human OC networks

Weight (number of edges) distributions for large human OC networks

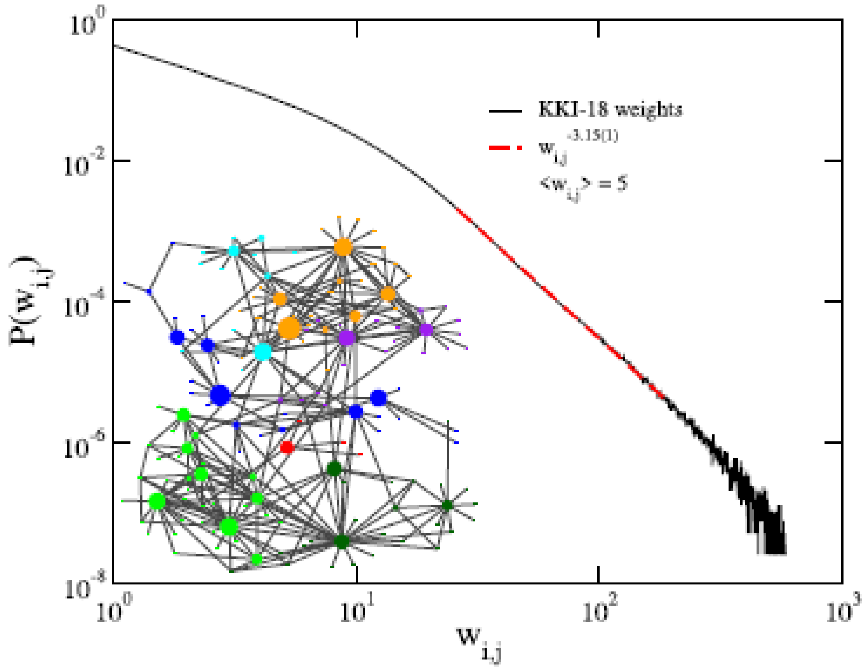


KKI-2009-18

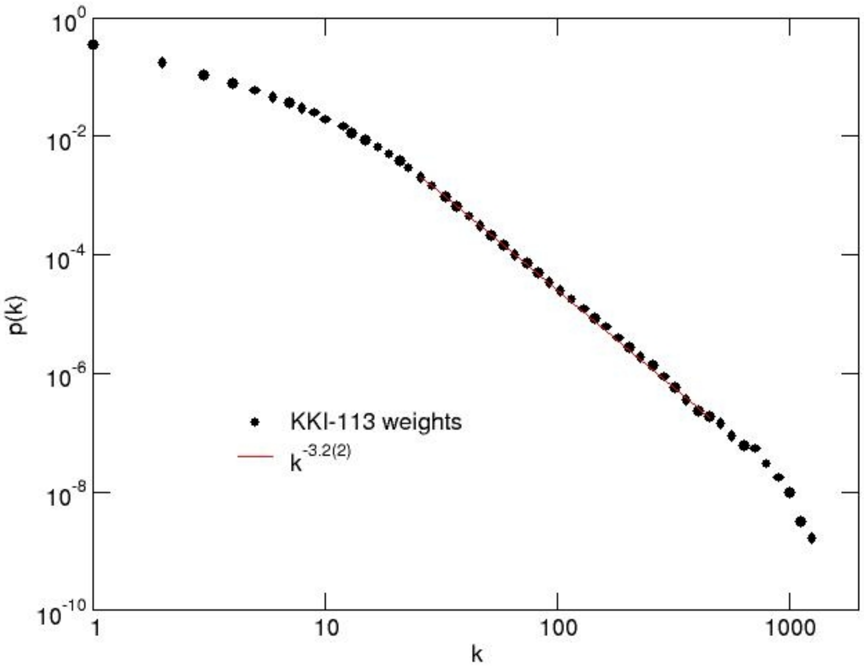
Géza Ódor et al 2021

J. Phys. Complex. 2 (2021) 045002

Weight (number of edges) distributions for large human OC networks



KKI-2009-18

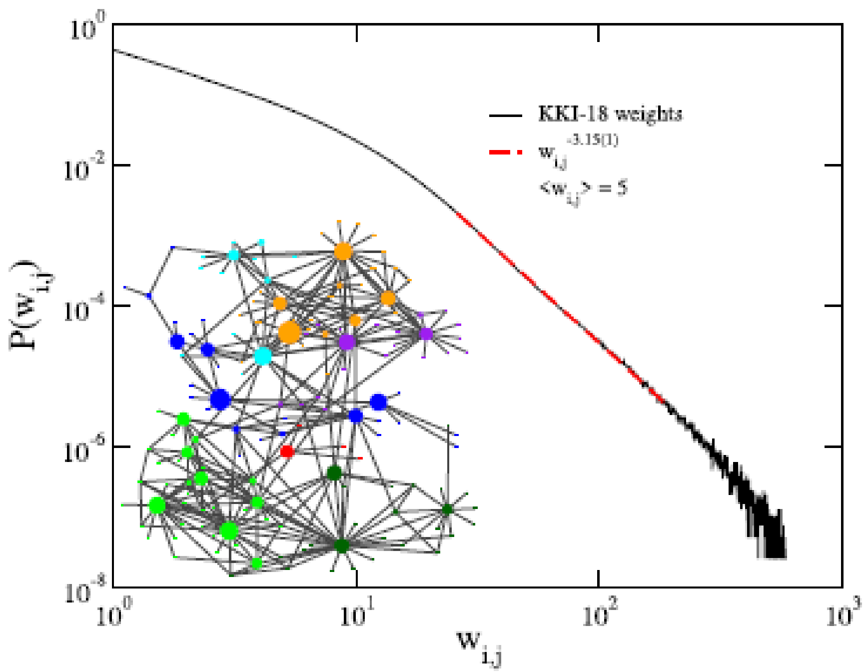


KKI-2009-113

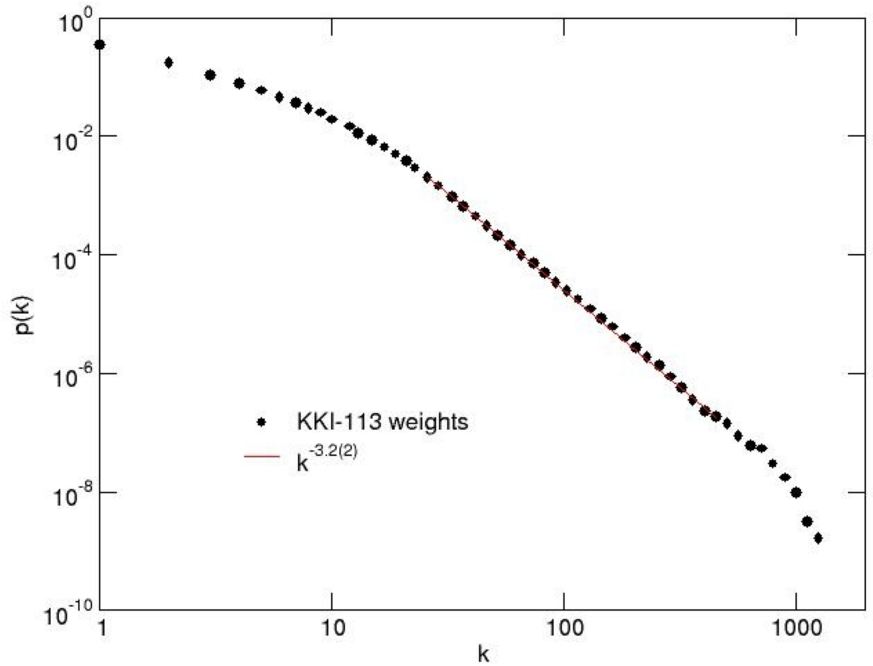
Géza Ódor et al 2021

J. Phys. Complex. 2 (2021) 045002

Weight (number of edges) distributions for large human OC networks



KKI-2009-18



KKI-2009-113

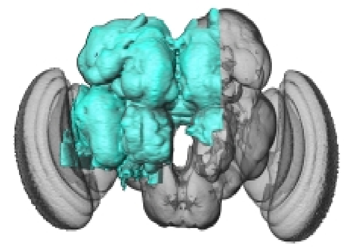
Géza Ódor et al 2021

J. Phys. Complex. 2 (2021) 045002

Power-law tails with exponents ~ *3*

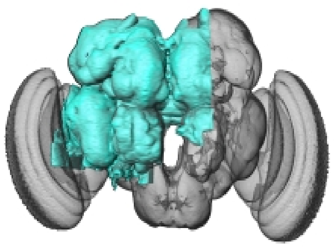
In-out strength pdf-s : faster than PL decay

Fruit-fly degree distributions

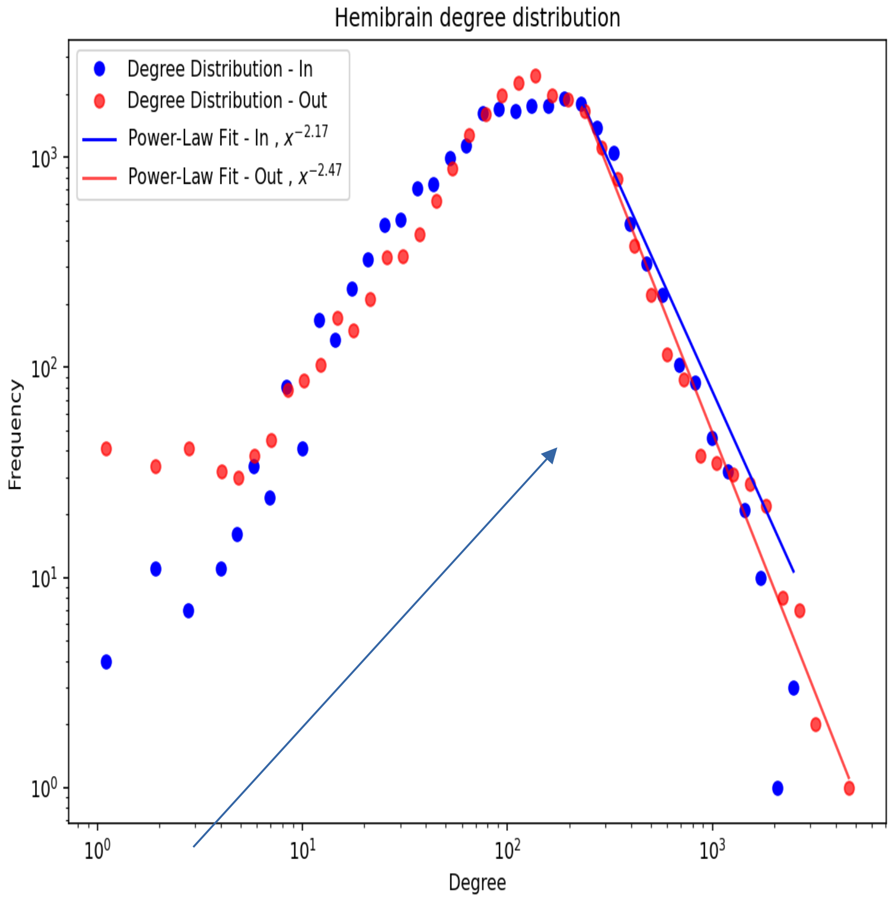


21.662
vs
124.891
nodes

Fruit-fly degree distributions

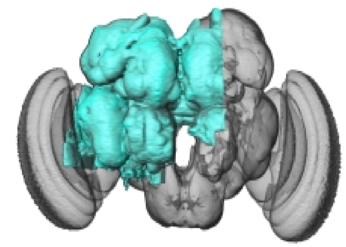


21.662
vs
124.891
nodes

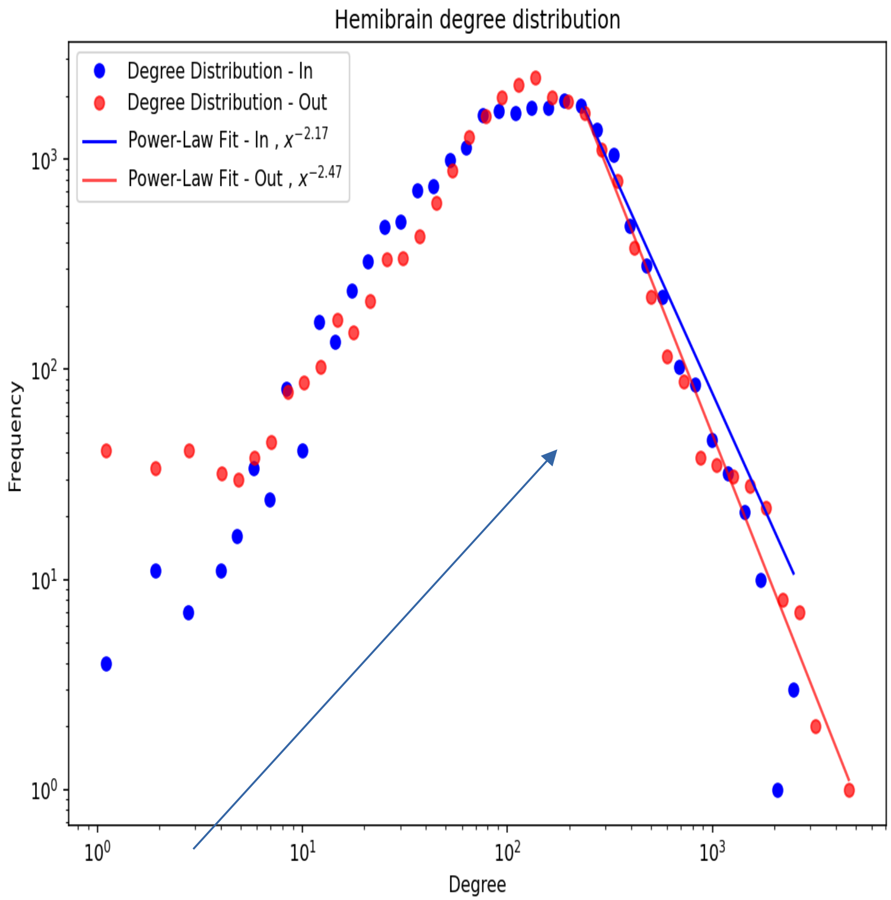


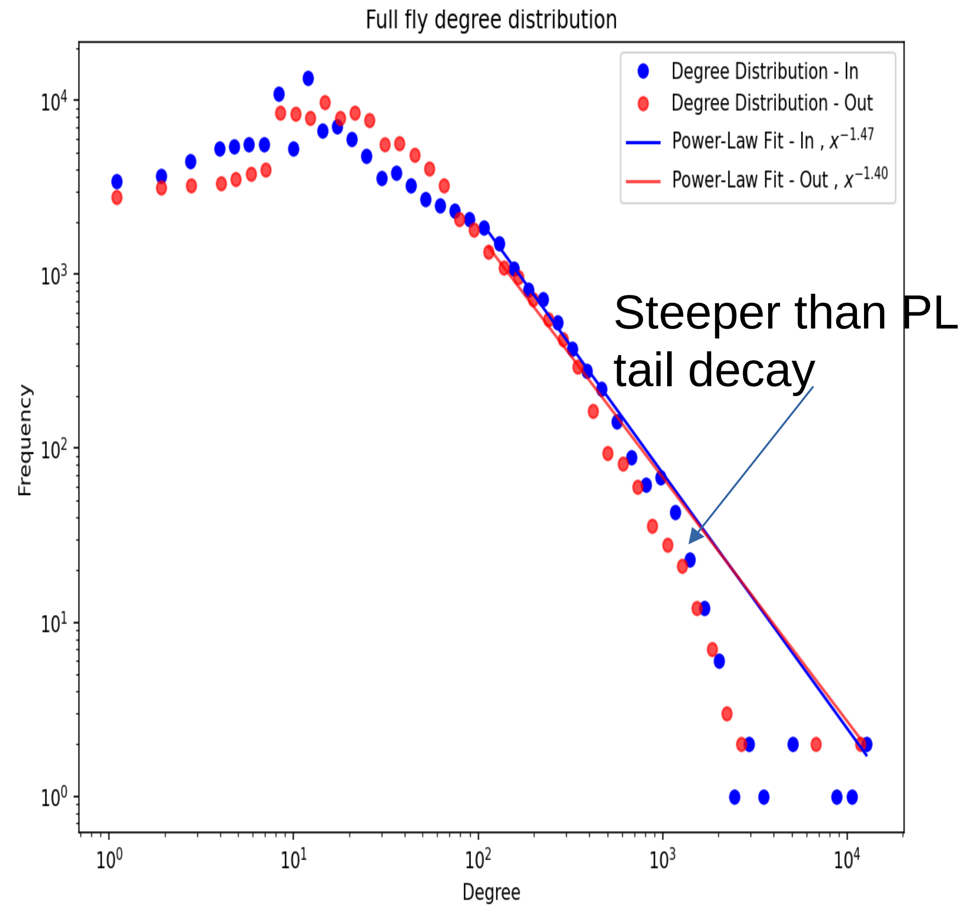
PL tail was fitted *(I.A. Kovacs et al)*

Fruit-fly degree distributions



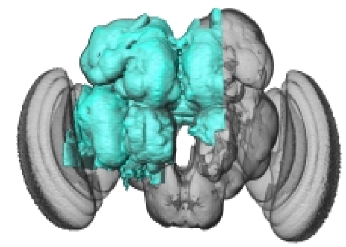
21.662
vs
124.891
nodes



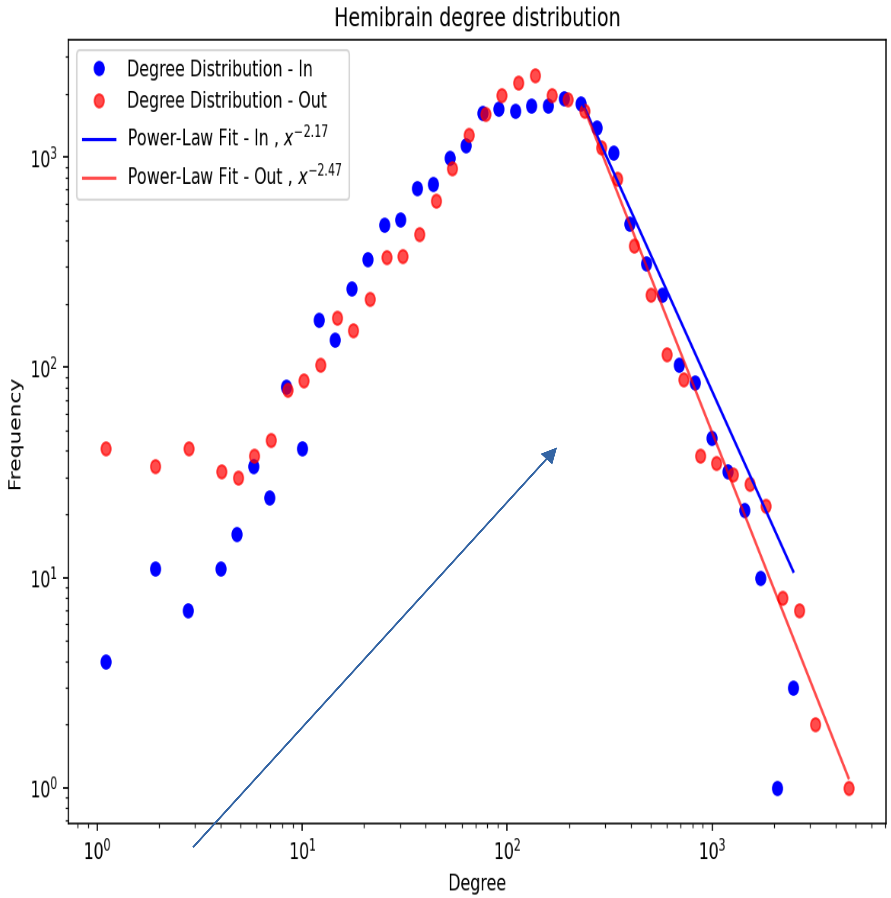


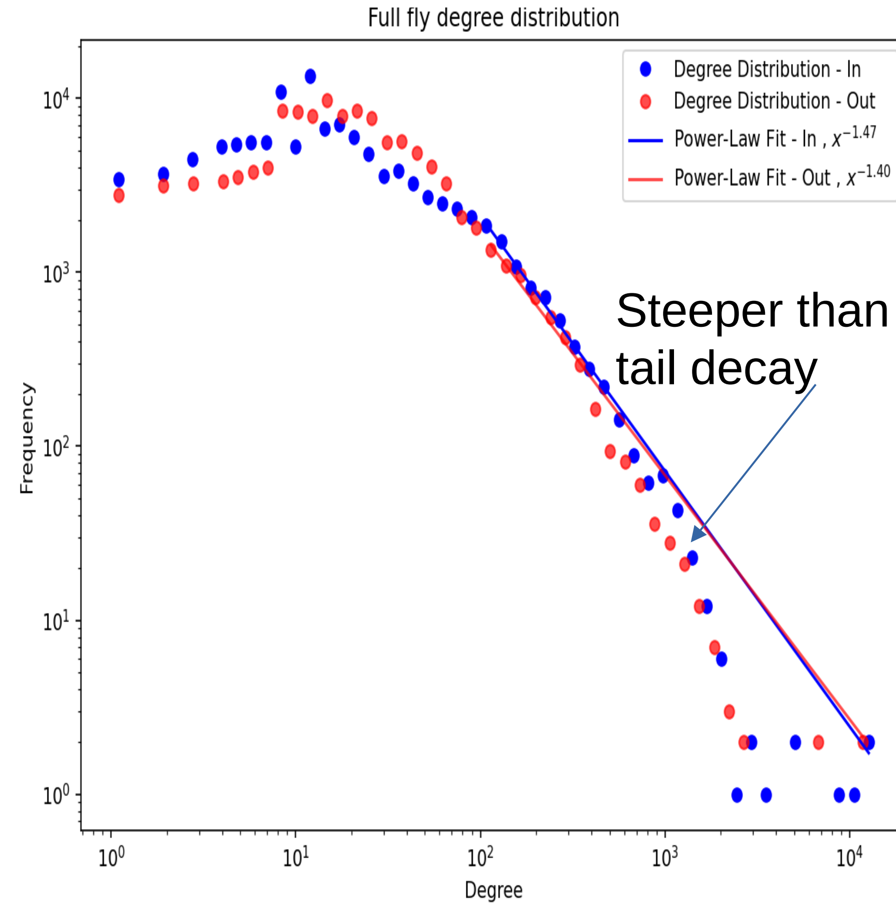
PL tail was fitted *(I.A. Kovacs et al)*

Fruit-fly degree distributions



21.662
vs
124.891
nodes



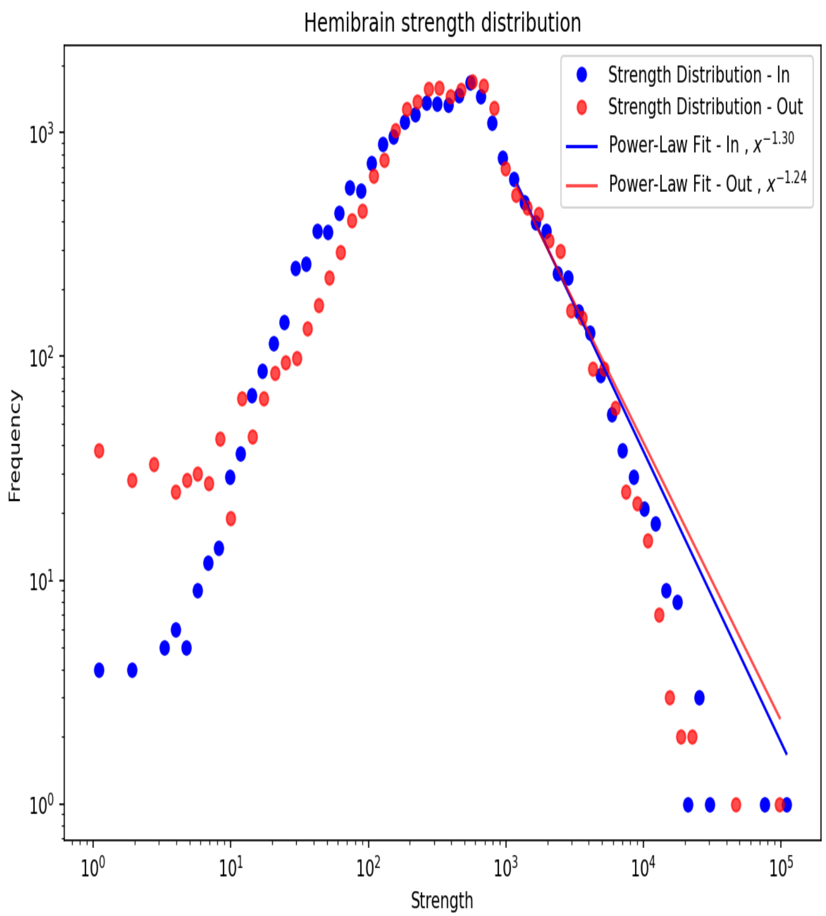


Steeper than PL tail decay

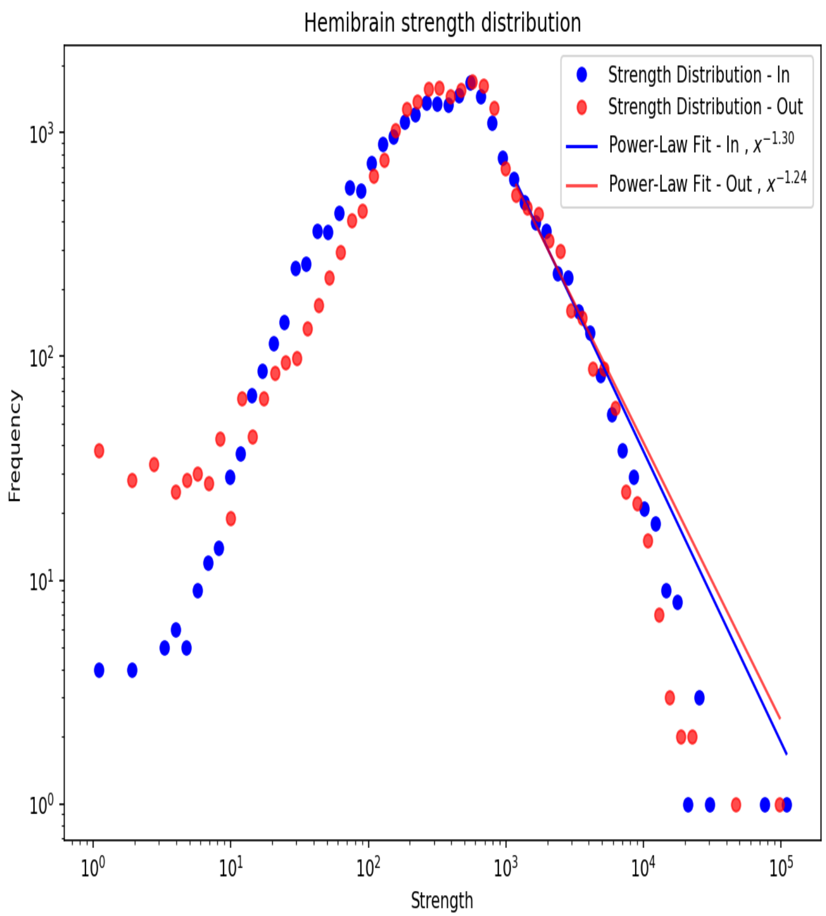
PL tail was fitted *(I.A. Kovacs et al)*

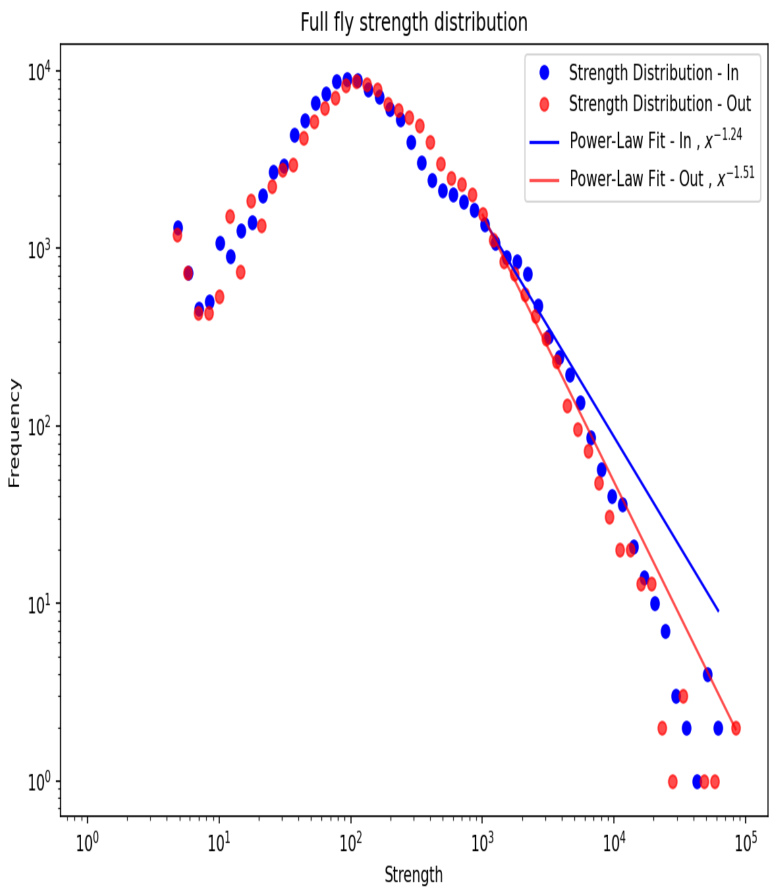
For the full fly brain the PL degree distribution fit breaks down

Fly node strengths (weight) distributions

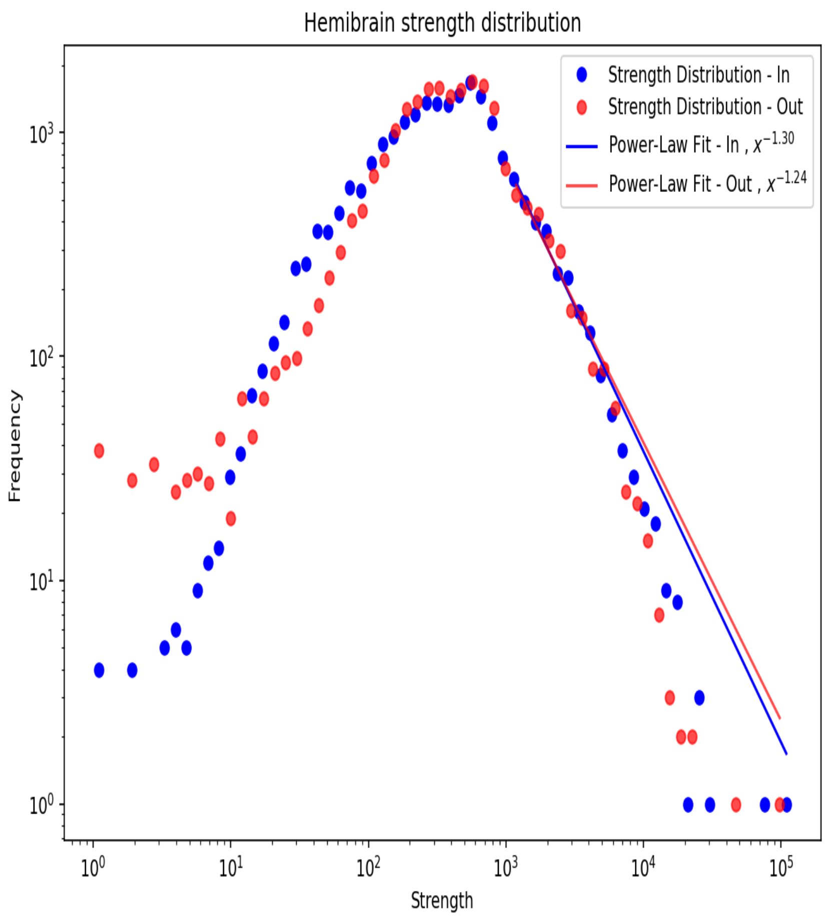


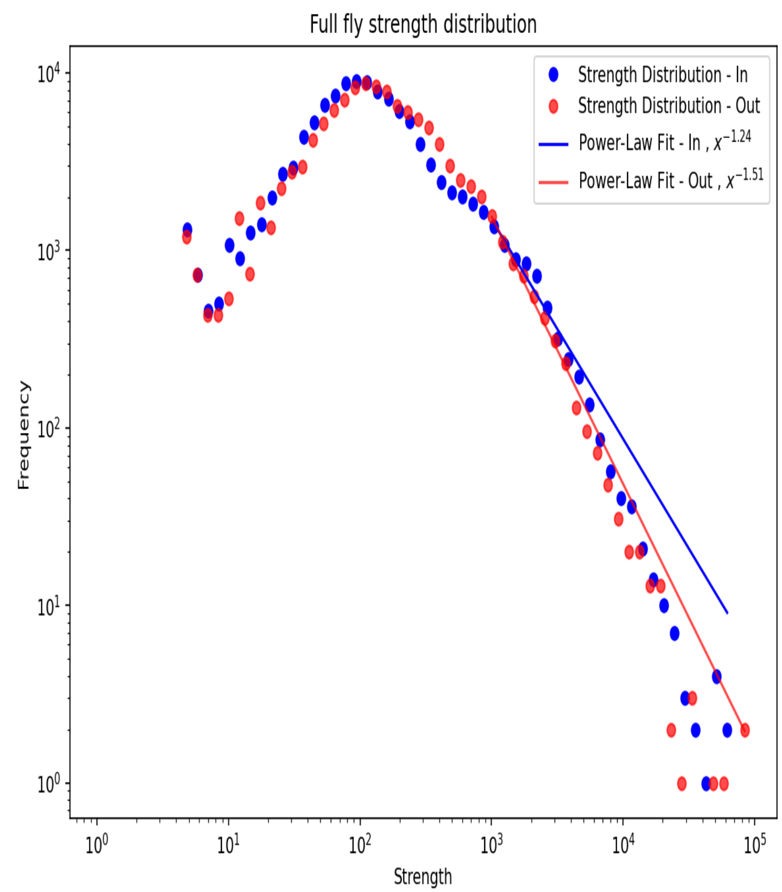
Fly node strengths (weight) distributions





Fly node strengths (weight) distributions





Fat tails, but faster than PL decay

Distribution of synapses (connection strengths weights) of the whole graph

Hemibrain

Full fly

PL tails with exponent ~3

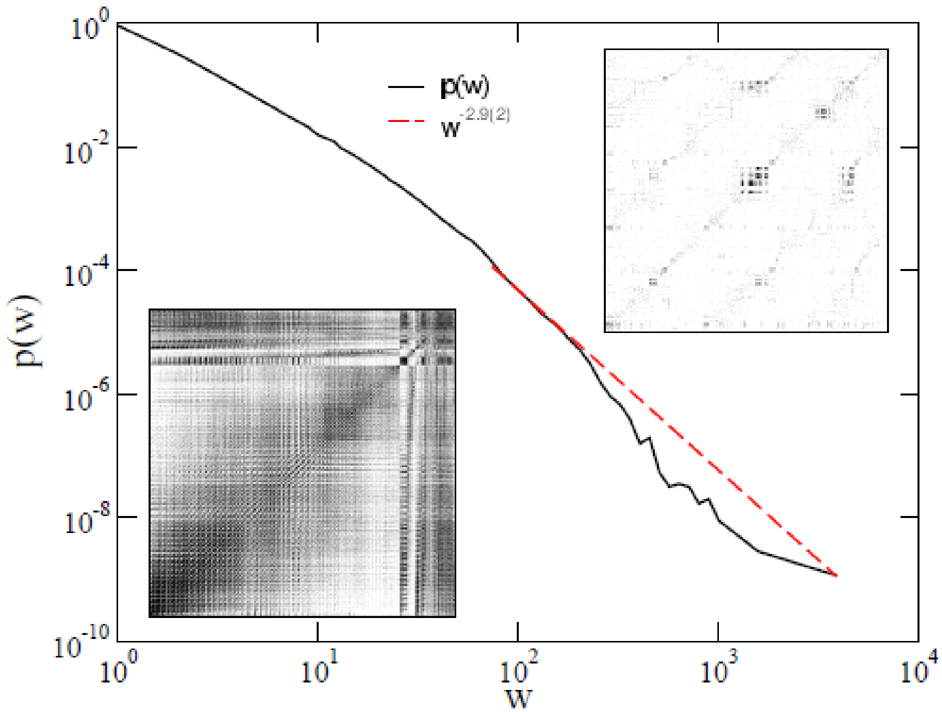
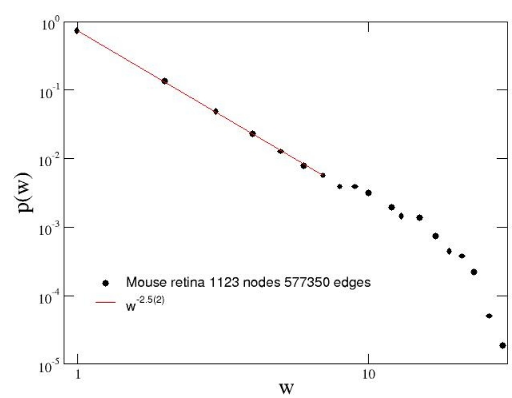


FIG. 1: Weight distribution of the fruit-fly connectome. Right inset: adjacency matrix plot of the fruit-fly connectome. Left inset: full adjacency matrix down-sampled with a max pooling kernel of size 10 × 10. Black dots denote connections between presynaptic and postsynaptic neurons. Right inset: zoom-in to the center of the matrix without down-sampling.



Mouse retina

G.Ó. et al Phys. Rev. Res. 4 (2022) 023057.

Distribution of synapses (connection strengths weights) of the whole graph

Hemibrain

Full fly

PL tails with exponent ~3

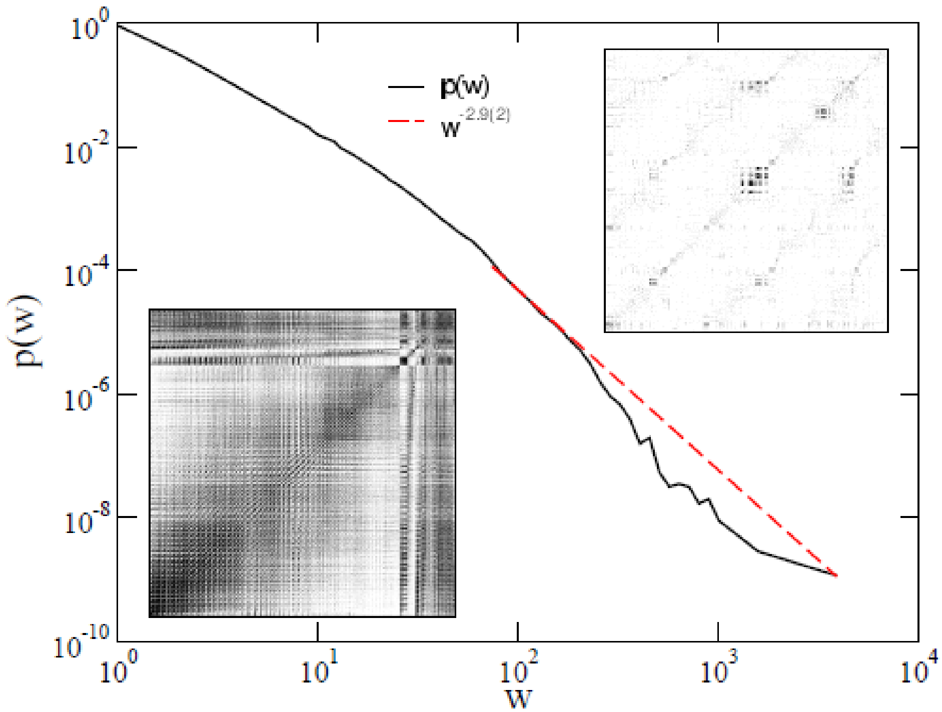
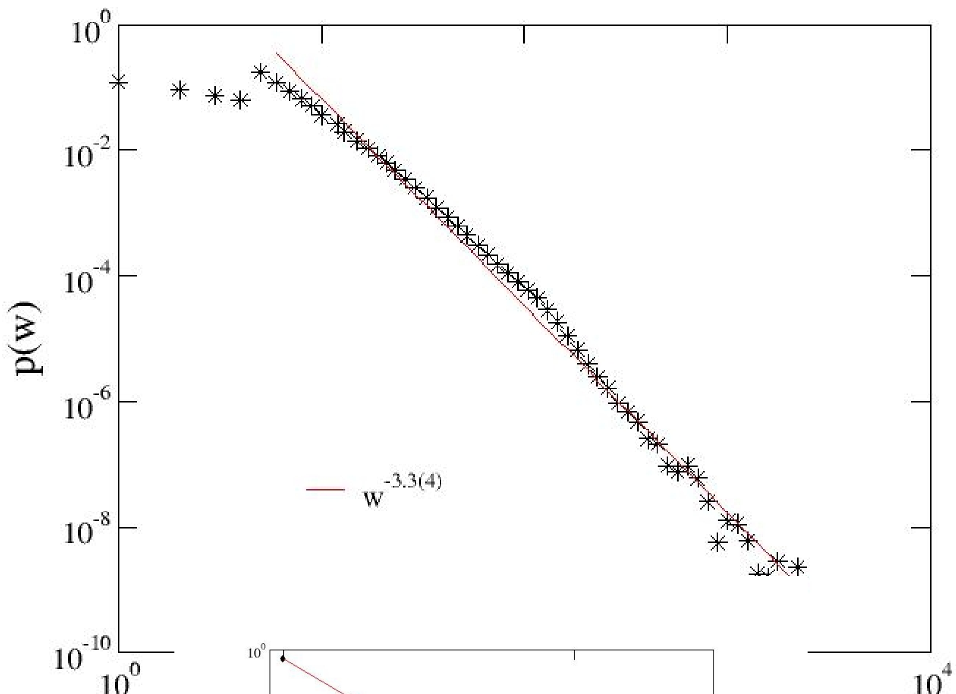
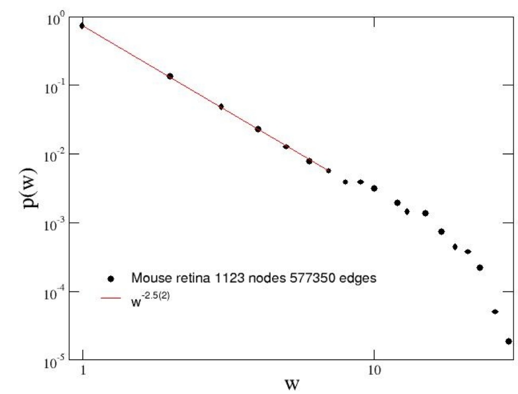


FIG. 1: Weight distribution of the fruit-fly connectome. Right inset: adjacency matrix plot of the fruit-fly connectome. Left inset: full adjacency matrix down-sampled with a max pooling kernel of size 10 × 10. Black dots denote connections between presynaptic and postsynaptic neurons. Right inset: zoom-in to the center of the matrix without down-sampling.

G.Ó. et al Phys. Rev. Res. 4 (2022) 023057.





Mouse retina

Conclusions

Conclusions

Node degree and weights distributions of large connectomes (human white matter), fly axon exhibit fat tails, but faster than power-laws

Conclusions

Node degree and weights distributions of large connectomes (human white matter), fly axon exhibit fat tails, but faster than power-laws

Global weight degree distributions can be fitted with PL tails of exponent: ~*3*

Conclusions

Node degree and weights distributions of large connectomes (human white matter), fly axon exhibit fat tails, but faster than power-laws

Global weight degree distributions can be fitted with PL tails of exponent: ~*3*

This suggests a non-local learning mechanism: rewiring that follows the critical dynamics of the brain function

Conclusions

Node degree and weights distributions of large connectomes (human white matter), fly axon exhibit fat tails, but faster than power-laws

Global weight degree distributions can be fitted with PL tails of exponent: ~*3*

This suggests a non-local learning mechanism: rewiring that follows the critical dynamics of the brain function

Géza Ódor, Michael T. Gastner, Jeffrey Kelling and Gustavo Deco
Modelling on the very large-scale connectome
J. Phys. Complex. 2 (2021) 045002.

Thank you for your attention !

Human in-out weights