Critical behavior of binary production reaction-diffusion systems

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Abstract. Recently reaction-diffusion type of models were introduced that involve particle (or pair) production and annihilation at pairs, while lonely particles follow annihilating random walks, hence they may be described by a system with a coupled diffusive field. Different numerical studies have found that this kind of models in 1d exhibit continuous phase transitions to absorbing states with new type of universal behavior for which neither bosonic field theoretical treatment nor arguments based on symmetries provided full understanding. Site restricted (fermionic) and unrestricted (bosonic) versions of these models exhibit different critical behaviors. In 2d large scale simulations (using grid network) show that mean-field type of behavior (with logarithmic corrections) emerges. In 1d it is debated whether the critical behavior changes by varying the diffusion rate or not. I show some recent simulation results in one and two dimensions that clarifies the situation to some extent. I give an outlook to generalizations of these models.

OVERVIEW OF DYNAMIC UNIVERSALITY CLASSES OF REACTION-DIFFUSION SYSTEMS

In complex, interacting systems built up from particles, galaxies, animals, cars, money, web nodes ... etc. power-laws and fractals are common. The set of power-law exponents define universality classes which are determined solely by the collective behavior of the parts. The most important and well known universality classes appear at critical phase transitions, where the correlation length diverges. In equilibrium systems these classes are well understood and they are tabulated in textbooks. The role of symmetries, dimensions, boundary conditions that affect these classes has been explored. In nonequilibrium systems similar scenario is hoped [1, 2], however recently novel classes without special symmetries or conservation laws have been discovered (see [3]).

Dynamical universality classes

The first step towards the study of nonequilibrium systems was the extension of static equilibrium classes (Ising, Potts, O(N)...) with different dynamics [4].

- non-conserving (model-A, Glauber),
- conserving (model-B, Kawasaki),
- model-C, ...

These kind of dynamics split up static classes and the introduction of dynamic exponents were needed to describe them. In the course of the research new phenomena like coarsening, initial slip, damage spreading ... etc. have also emerged in such dynamical systems.

Later mixture of different types of dynamics were introduced to create out of equilibrium systems. Such systems are characterized by the broken detailed balance (global time reversal) condition of the transition rates ($w_{i \to j}$)

$$w_{i\to j} P(s(i)) = w_{j\to i} P(s(j)) \ , \tag{1}$$

where $P(s(i))$ is the probability distribution of states $s(i)$. It was shown that most of the systems investigated exhibit the original dynamic class behavior or long range interaction classes are generated by adding a Kawasaki particle exchange (for recent review see [5]). However if we mix a competing dynamics to model-B systems novel kind of universal behavior has also been found [6, 7]. It is also important that in nonequilibrium systems phase transitions may happen in very low dimensions and the Landau-Peierls, Mermin-Wagner theorems are not valid in general.

Reaction-diffusion systems

Phase transitions in genuine nonequilibrium systems have been investigated mostly among reaction-diffusion (RD) types of models exhibiting absorbing states [1, 2, 3]. In many cases mapping onto surface growth, spin systems or stochastic cellular automata can be done. Therefore they can be considered as main representatives of general, nonequilibrium models. In these systems usually diffusion, production and particle annihilation compete with each other and may result in phase transitions to absorbing state(s), from which no return is possible owing to the small level of fluctuations.

Starting such a system from a single active state at criticality (σ_c) the following scaling quantities are followed usually:

$$\rho(t) \propto t^{\eta}, \quad P(t) \propto t^{-\delta}, \quad R^2(t) \propto t^{z}, \tag{2}$$

where ρ denotes the particle density, P the survival probability of a growing cluster and R measures the mean spread of distance from the origin. Later correlations build up and in case of overlapping clusters the density increase crosses over to density decay characterized by

$$\rho(t) \propto t^{-\alpha} \ . \tag{3}$$

Such phenomena is called "critical initial slip" suggested first by Janssen et al. [8]. In the active phase ($\sigma > \sigma_c$) and in the infinite time limit the cluster survival probability P is expected to scale as

$$P(\sigma) \propto (\sigma - \sigma_c)^{\beta'} \ , \tag{4}$$

while the density of active sites as

$$\rho(\sigma) \propto (\sigma - \sigma_c)^{\beta} \ . \tag{5}$$

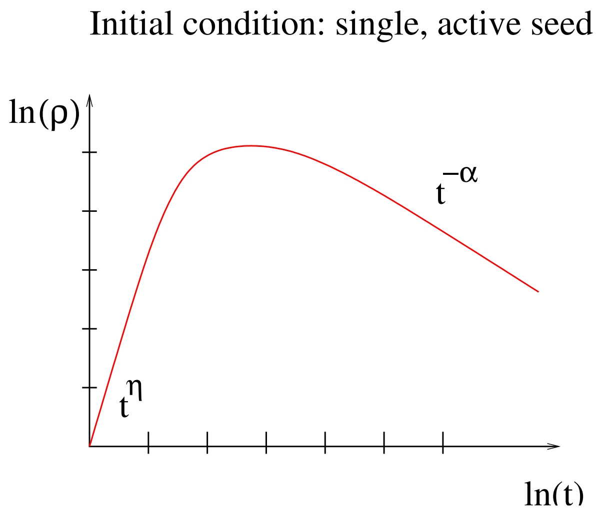

FIGURE 1. Evolution of particle density in RD systems

Field theoretical approach to reaction-diffusion systems (ARW)

In the following sections I shall overview the most well known classes of absorbing state transitions in field theoretical formalism. For this reason in this subsection I give a brief introduction to (bosonic) field theoretical formalism . This will be done through the simplest reaction-diffusion system the: $A+A \to \emptyset$ annihilating random walk (ARW) [9].

A proper field theoretical treatment should start from the master equation for the microscopic time evolution of probabilities $p(\alpha;t)$ of states α

$$\frac{dp(\alpha;t)}{dt} = \sum_{\beta} R_{\beta\to\alpha} p(\beta;t) - \sum_{\beta} R_{\alpha\to\beta} p(\alpha;t) , \tag{6}$$

where $R_{\alpha\to\beta}$ denotes the transition matrix from state α to β. In field theory this can be expressed in Fock space formalism with annihilation (a_i) and creation (c_i) operators satisfying

$$[a_i, c_j] = \delta_{ij} . \tag{7}$$

The states are built up from the vacuum $|0>$ as the linear superposition

$$\Psi(t) = \sum_{\alpha} p(n_1, n_2, ...; t) c_1^{n_1} c_2^{n_2} ... |0> , \tag{8}$$

with occupation number coefficients $p(n_1, n_2, ...; t)$. The evolution of states can be described by a Schrödinger-like equation

$$\frac{d\Psi(t)}{dt} = -H\Psi(t) \tag{9}$$

with a generally non-hermitian Hamiltonian, which in case of the ARW process looks like

$$H = D\sum_{ij}(c_i - c_j)(a_i - a_j) - \lambda \sum_j (a_j^2 - c_j^2 a_j^2) \; . \tag{10}$$

By going to the continuum limit this turns into

$$H = \int d^d x \left[D(\nabla\psi)(\nabla\phi) - \lambda(\phi^2 - \psi^2\phi^2) \right] \; , \tag{11}$$

and in the path integral formalism over fields $\phi(x,t)$, $\psi(x,t)$ with weight $e^{-S(\phi,\psi)}$ one can define an action, that in case of ARW is

$$S = \int dt d^d x \left[\psi\partial_t\phi + D\nabla\psi\nabla\phi - \lambda(\phi^2 - \psi^2\phi^2) \right] \; . \tag{12}$$

The action is analyzed by renormalization group methods at criticality, usually by perturbative epsilon expansion below the upper critical dimension d_c — that is the lower limit of the validity of the mean-field behavior of the system. The symmetries of the model can be expressed in terms of the $\phi(x,t)$ field and $\psi(x,t)$ response field variables and the corresponding hyperscaling laws can be derived [10].

By a Gaussian transformation one can set up an alternative formalism — integrating out the response field — the Langevin equation, that in case of ARW is

$$\partial_t\phi = D\nabla^2\phi - 2\lambda\phi^2 + \eta(x,t) \tag{13}$$

with a Gaussian noise term satisfying:

$$< \eta(x,t)\eta < x',t') >= -\lambda\phi^2\delta(x-x')\delta(t-t') \; . \tag{14}$$

Genuine nonequilibrium classes

In this section I briefly summarize the basic universality classes appearing at phase transitions to absorbing states.

The DP class

The most extensively studied, robust universality class is the directed percolation (DP) [11], which appears canonically in RD systems with particle creation

$$A \stackrel{\sigma}{\to} 2A \; , \tag{15}$$

particle coagulation

$$2A \stackrel{\lambda}{\to} A \; , \tag{16}$$

and single particle self-destruction

$$A \stackrel{\kappa}{\to} \emptyset. \tag{17}$$

Notice, that particle diffusion and higher order terms are also generated by these processes. By tuning the rates this model exhibits a continuous phase transition to a single, empty absorbing state. The critical behavior is discussed for example in [2]. One can set up the field theoretical action

$$S = \int dt d^d x \left[\psi(\tau \partial_t - D\nabla^2 - \kappa)\phi + \Gamma/2 \psi(\phi - \psi)\phi \right] \tag{18}$$

that exhibits the rapidity inversion symmetry:

$$\phi(x,t) \to -\psi(x,-t), \qquad \psi(x,t) \to -\phi(x,t) \tag{19}$$

and as the consequence the hyperscaling law

$$4\delta + 2\eta = dz \tag{20}$$

among dynamical exponents holds. The upper critical dimension of this field theory is $d_c = 4$. This class has been observed in many model systems and the DP hypothesis [12, 13] was set up. This DP hypothesis claims that in one component systems exhibiting continuous phase transitions to single absorbing state (without extra symmetry and inhomogeneity or disorder) short ranged interactions can generate DP class transition only. Despite the robustness of this class experimental observation is still lacking [14] probably owing to the sensitivity to disorder that can not be avoided in real materials.

The PC class

In branching and annihilating random walks with even number of offsprings (BARWe) an other universality class appears. The canonical processes are the followings. Particle pair creation

$$A \stackrel{\sigma}{\to} 3A \,, \tag{21}$$

particle pair annihilation

$$2A \stackrel{\lambda}{\to} \emptyset \,, \tag{22}$$

and single particle diffusion

$$A\emptyset \stackrel{D}{\to} \emptyset A. \tag{23}$$

This process conserves the particle number modulo 2, therefore from the beginning its phase transition class has been called parity conserving (PC) [15, 16] owing to this law that differentiated it from the robust DP class. This class has also been observed in models exhibiting Z_2 symmetric absorbing states, where domain walls separating ordered phases follow BARWe dynamics [17, 18, 19, 20, 21, 22]. The field theoretical action was set up by Cardy and Täuber [23]

$$S = \int d^d x dt \left[\psi(\partial_t - D\nabla^2)\phi - \lambda(1-\psi^2)\phi^2 + \sigma(1-\psi^2)\psi\phi \right] \tag{24}$$

but unfortunately systematic ε expansion renormalization group analysis run into problems and did not give quantitatively precise exponent estimates. This is the consequence of two critical dimensions: $d_c = 2$ and $d_c' = 4/3$ found in the model. The action is invariant under the simultaneous transformation of fields

$$\psi \leftrightarrow -\psi, \quad \phi \leftrightarrow -\phi \tag{25}$$

Owing to the non-recurrence of random walks in $d \geq 2$ the system is in the active phase for $\sigma > 0$ and mean-field type transition occurs with $\beta = 1$. However the survival probability of a particle cluster is finite for any $\sigma > 0$ implying $\beta' = 0$. Hence unlike the DP case $\beta \neq \beta'$ for $d \geq 2$. At $d = 2$ random walks are barely recurrent and logarithmic corrections can be found. In this case the generalized hyperscaling law [24] is valid among the exponents

$$2\left(1+\frac{\beta}{\beta'}\right)\delta + 2\eta = dz. \tag{26}$$

In $d = 1$ however $\beta = \beta'$ holds owing to an exact duality mapping [25] and the hyperscaling is the same that of the DP (eq. 20).

The Dynamical percolation (DyP) class

The dynamical percolation is defined by similar processes as the DP but the spreading agent has memory (no re-infection). The field theoretical action is [26, 27, 28, 29]

$$S = \int d^dxdt\left[\frac{D}{2}\psi^2\phi - \psi\left(\partial_t\phi - \nabla^2\phi - r\phi + w\phi\int_0^t ds\phi(s)\right)\right] \tag{27}$$

which exhibits the symmetry

$$\phi(x,t) \leftrightarrow -\partial_t\psi(x,t) \tag{28}$$

and as the consequence the hyperscaling law

$$\eta + 2\delta + 1 = \frac{dz}{2} \tag{29}$$

holds. In d>1 it generates isotropic percolation clusters and the upper critical dimension is $d_c = 6$.

The Voter model (VM) class

The voter model [30, 31] is defined on spin systems exhibiting activity at boundaries only. This ensures that the model has two homogeneous absorbing states (all spin up or down) and invariant under Z_2 symmetry. The field theoretical action is [32, 33].

$$S = \int d^dxdt\left[\frac{D}{2}\phi(1-\phi)\psi^2 - \psi(\partial_t\phi - \lambda\nabla^2\phi)\right], \tag{30}$$

which is invariant under the transformation

$$\phi \leftrightarrow 1-\phi, \quad \psi \leftrightarrow -\psi \quad . \tag{31}$$

and as the consequence the hyperscaling law

$$\delta + \eta = dz/2 \tag{32}$$

is satisfied. It is valid for all first order transitions ($\beta = 0$) with $d \leq 2$, hence $d_c = 2$ is the upper critical dimension.

Genuine nonequilibrium classes in multi-component systems

Besides one-component systems, that are quite well established in the recent years novel classes have been identified in multi-component systems. However their behavior is still the target of intensive research. Here I mention some of them in which DP process is coupled to some other mode following annihilating random walks.

DP coupled with conserved diffusive field

The first example was suggested by Kree et al [34] and Wijland et al [35]. In this case two types of particles A-s and B-s execute the following processes

$$A + B \to 2B, \qquad B \to A \ . \tag{33}$$

The order parameter is the density of B-s and the control parameter it the total number of particles that is conserved. The field theory was set up by Wijland et al. [35] and different, novel type of transitions were predicted for: $D_A < D_B$, $D_A = D_B$, $D_A > D_B$.

DP coupled with a conserved frozen field

These models are similar to the previous ones but here the A particles do not move alone, they just provide a "background field" for B-s [36, 37, 38, 39, 40]. Examples are the conserved lattice gas, CTTP and LIM models. Numerical results exist for $1 < d < 5$.

DP coupled with a frozen non-diffusive field

The pair contact process: $2A \to 3A$, $2A \to \emptyset$ introduced by Dickman and Jensen [41] can also be considered as a two component system, in which pairs follow DP process, while lonely particles just provide a background field, and destroy the rapidity inversion symmetry of DP. As the consequence the generalized hyperscaling is valid for dynamical exponents. The critical behavior is DP like, but in case of inhomogeneous (seed) initial conditions non-universal scaling of $\rho(t)$ was found.

DP coupled with a diffusive field

This family of models was found to be related to binary production PCPD models [42] by a mapping (see next section). By generalizing the processes these models may also be related to triplet or quadruplet production models, that have been introduced recently [43, 44, 45].

The binary production PCPD model

The diffusive pair contact process (PCPD) introduced by Carlon et al. [46] is controlled by two independent parameters: the probability of pair annihilation p and the probability of particle diffusion D. The dynamic rules are

$$\begin{aligned} AA\emptyset, \emptyset AA \to AAA &\quad \text{with rate} \quad (1-p)(1-D)/2 \\ AA \to \emptyset\emptyset &\quad \text{with rate} \quad p(1-D) \\ A\emptyset \leftrightarrow \emptyset A &\quad \text{with rate} \quad D\,. \end{aligned} \tag{34}$$

The model exhibits two absorbing states without symmetry, one of them is diffusive. Generalized, *cluster mean-field* approximation was applied for the dynamical rules (34) of the 1d fermionic lattice model (see Fig.2). It was shown in [47] that at least $N > 2$ level of approximation is necessary to obtain a correct mean-field behavior.

The bosonic field theory [48] failed to describe critical and supercritical behavior because in the bosonic model the density diverges in the active phase. The site restricted model shows different critical behavior but a corresponding fermionic field theory has not been solved yet. The action of the coarse-grained bosonic version is

$$S = \int d^dxdt \left[\psi(\partial_t - D\nabla^2)\phi - \lambda(1-\psi^2)\phi^2 + \sigma(1-\psi^2)\psi^2\phi^2\right] \,. \tag{35}$$

Numerical methods have shown novel type of critical exponents [46, 49, 50, 42, 51, 52, 53, 54], but no extra symmetries or conservation laws have been found. However complex noise correlator in the Langevin equations is necessary to describe this model.

NUMERICAL RESULTS FOR BINARY PRODUCTION REACTION-DIFFUSION MODELS

In this section I show some of the latest extensive simulation and cluster mean-field results.

Recent PCPD simulation results in 1+1 d

Simulations in 1d were performed on $L = 10^5$ sized rings with random sequential update version of PCPD up to $t = 10^8$ MCS. The number of particles (N_p) is followed

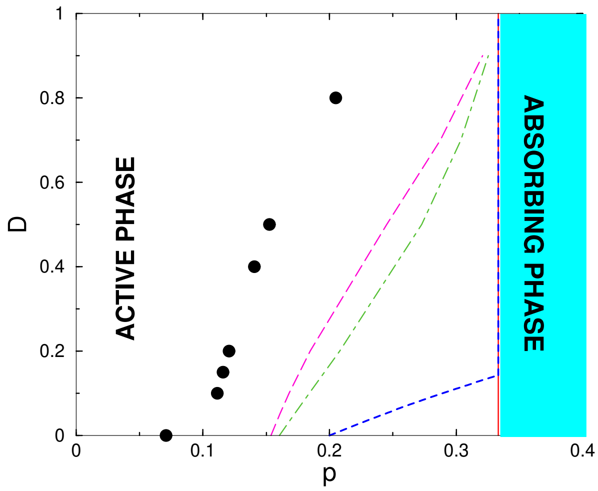

FIGURE 2. The phase diagram of the 1 d PCPD model determined by simulations and DMRG method (bullets) and by different levels of cluster mean-field approximations: (solid line $N=1$), (dashed line $N=2$), (dot-dashed line $N=3$) and (long-dashed line $N=4$).

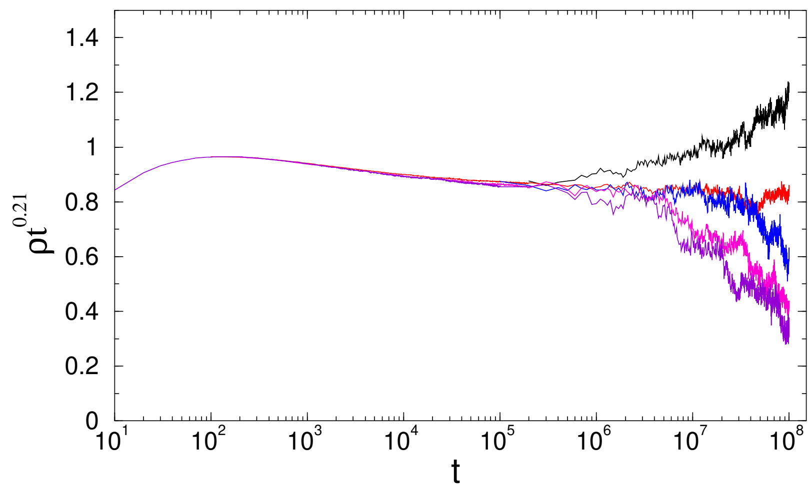

FIGURE 3. Density decay times $t^{0.21}$ in 1d PCPD at $D=0.2$ and $p=0.111215, 0.11217, 0.11218, 0.11219, 0.1122$ (top to bottom).

and the time is updated by $1/N_p$ following a reaction (throughout the whole paper the time is measured by Monte Carlo steps (MCS)). The initial conditions were random distribution of particles with an occupation probability 0.5. The density decay of particles was followed at diffusion rate $D=0.2$ (Fig.3) and at $D=0.5$ (Fig.4). On all plots one can see up and down veering $\rho(t)$ curves in the long time limit – corresponding to active and absorbing phases – separated by a roughly straight line – corresponding to p_c. On can read-off decay exponents in both cases: $\alpha=0.21(1)$. However for $D<0.2$ the scaling exponent seems to be somewhat smaller $\alpha=0.245(4)$ suggesting two distinct universality classes.

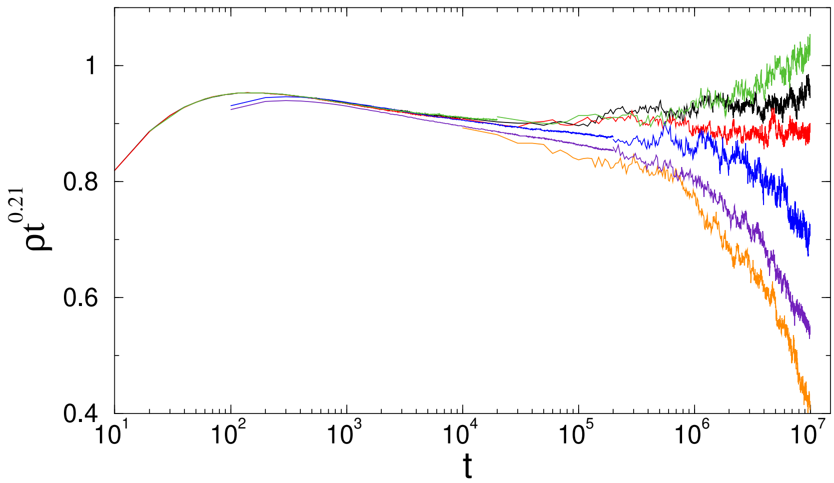

FIGURE 4. Density decay times $t^{0.21}$ in 1d PCPD at $D = 0.5$ and $p = 0.13351, 0.13352, 0.13353, 0.13356, 0.1336, 0.13363$ (top to bottom).

To estimate directly the order parameter exponent, which describes the scaling

$$\rho(\infty, \varepsilon) \propto \varepsilon^{\beta} \tag{36}$$

off-critical, steady state densities were measured precisely. The density decay had been followed for each D and $\varepsilon_i = p_c - p_i$ values on logarithmic time scales until saturation effect was observed. Following that averaging of $\rho(t)$ was done for about 100 samples within a time window that exceeds the saturation by a decade. The effective exponents defined as

$$\beta_{eff} = \frac{\ln(\rho(\infty, \varepsilon_i)) - \ln(\rho(\infty, \varepsilon_{i-1}))}{\ln(\varepsilon_i) - \ln(\varepsilon_{i-1})} \ , \tag{37}$$

which are expected to converge to the true critical values in the $\varepsilon \to 0$ limit were determined. As one can see on Fig.5 the local slopes for $D = 0.7$ and $D = 0.5$ converge to $\beta = 0.40(1)$ in agreement with the high diffusion rate results provided in [50]. This value is also close to Hinrichsen's estimate $(0.38(6))$ for the cyclically coupled model [42] and to Kockelkoren's value $(0.37(2))$ for the suppressed bosonic SCA model [44].

In [47] I showed that it is possible to have a single universality class if we assume logarithmic corrections to scaling of the forms

$$\rho(t, p_c) = [(a + b\ln(t))/t]^{\alpha} \quad . \tag{38}$$

$$\rho(\infty, \varepsilon) = [\varepsilon/(a + b\ln(\varepsilon))]^{\beta} \tag{39}$$

The results are shown in Table 1.

Finite size scaling investigations at p_c were performed for system sizes $L = 32, 64, 128 \ldots 1024$. The quasi-steady state density (averaged over surviving samples) is expected to scale according to

$$\rho_s(\infty, p_c, L) \propto L^{-\beta/\nu_\perp}, \tag{40}$$

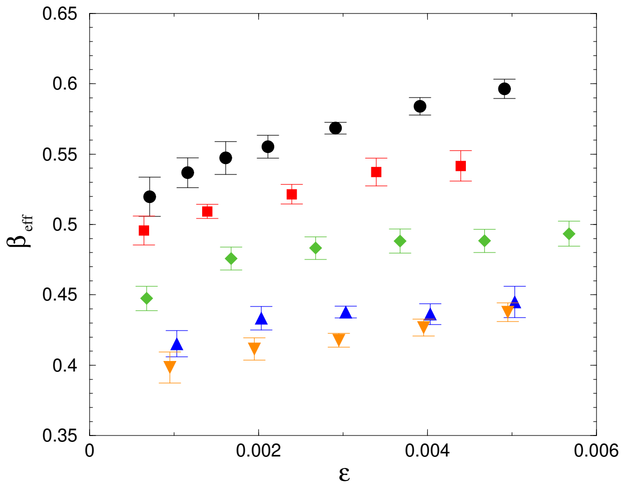

FIGURE 5. Effective β exponents for different diffusion rates. The circles correspond to $D = 0.05$, the squares to $D = 0.1$ the diamonds to $D = 0.2$, the up-triangles to $D = 0.5$ and the down-triangles to $D = 0.7$.

TABLE 1. Summary of simulation results for the 1d PCPD model assuming logarithmic corrections of the forms (38) and (39).

D	p_c	β	α
0.7	0.15745(1)	0.39(1)	0.214(5)
0.5	0.13353(1)	0.414(16)	0.206(7)
0.2	0.11218(1)	0.402(8)	0.217(8)
0.1	0.10688(1)	0.407(7)	0.206(7)
0.05	0.10439(1)	0.411(10)	0.216(9)

while the characteristic lifetime for half of the samples to reach the absorbing state scales with the dynamical exponent Z as

$$\tau(p_c, L) \propto L^Z . \tag{41}$$

Since the system sizes are much smaller than those in the α and β exponent simulations one may expect stronger corrections to scaling. Indeed the power-law fitting for $\beta/\nu_\perp$ results in values in the range $0.385 - 0.535$ and for Z in the range $1.75 - 2$ depending on D. These results are shown of Fig.6. Again the low-D data are in agreement with those of [46], [55] and [56], while the high-D data with those of [46], [44] and [42]. Just considering these ranges one can not distinguish this transition from the PC class (with $\beta/\nu_\perp = 0.500(5)$ and $Z = 1.75(1)$ [16]) that caused initial debates in the literature [46, 49, 50]. Assuming single universality class — corresponding to high-D data we may expect: $\beta/\nu_\perp = 0.38(1)$ and $Z = 1.75(15)$.

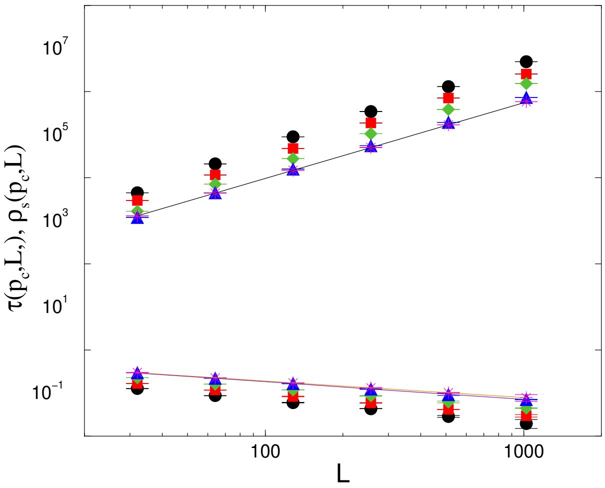

FIGURE 6. Finite size scaling of τ_L (upper points) and $\rho_L(\infty)$. The circles correspond to $D = 0.05$, the squares to $D = 0.1$ the diamonds to $D = 0.2$, the up-triangles to $D = 0.5$ and the down-triangles to $D = 0.7$. The lines show power-law fittings applied for $D = 0.7$ data points.

Related results

In this subsection I summarize the main results obtained for some related binary production systems in 1d recently.

The two-component, cyclically coupled DP+ARW model

Hinrichsen proposed [42] that one can map the PCPD model (similarly to the PCP case) onto a cyclically coupled, two component system such that pairs (A-s) follow a DP process,

$$A \leftrightarrow 2A, \quad A \to \emptyset\ , \tag{42}$$

while lonely particles (B-s) execute an annihilating random walk

$$2B \to A\ , \tag{43}$$

such that they create A-s and A-s can also transmutate to B-s. He has showed that the space-time evolution of this system is very similar to that of the PCPD model — compact domains surrounded by clouds of lonely particles — and exhibits a continuous phase transition at $D = 0.5$ with similar exponents as the PCPD model:

$$\alpha = 0.21(1), \qquad \beta = 0.38(6), \qquad 2/z = 1.75(5)\ . \tag{44}$$

The coagulation-production model

If we replace the annihilation in the PCPD model by a coagulation process:

$$\begin{aligned} AA\emptyset, \emptyset AA \to AAA \quad & \text{with rate} \quad (1-p)(1-D)/2 \\ AA \to A\emptyset \quad & \text{with rate} \quad p(1-D)/2 \\ AA \to \emptyset A \quad & \text{with rate} \quad p(1-D)/2 \\ A\emptyset \leftrightarrow \emptyset A \quad & \text{with rate} \quad D\,. \end{aligned} \tag{45}$$

we find a continuous phase transition in this system showing PCPD type of universal behavior [51, 55]. It was also shown in [51] that the one can replace the creations: $AA\emptyset \to AAA$, $\emptyset \to AAA$ with a symmetric one: $A\emptyset A \to AAA$ without changing the universal behavior. In the inactive phase of this model for the case $D = p(1-D)$ an exact solution was found for the density decay in 1d: $\rho \propto t^{-1/2}$ [57].

The parity conserving PCPD

Simulations have also provided numerical evidence [52, 55] that the parity conserving binary production process

$$\begin{aligned} AA\emptyset\emptyset, \emptyset\emptyset AA \to AAAA \quad & \text{with rate} \quad (1-p)(1-D)/2 \\ AA \to \emptyset\emptyset \quad & \text{with rate} \quad p(1-D) \\ A\emptyset \leftrightarrow \emptyset A \quad & \text{with rate} \quad D \end{aligned} \tag{46}$$

also shows the same kind of critical behavior as the PCPD model for a given diffusion strength D. This insensitivity to parity conservation is different from what we learned in case of unary production BARW models and should be a subject of further research.

The multi-component PCPD

This model was defined as a kind of binary production model with different types of particles [58]

$$AB \to ABA, \qquad AB \to BAB, \tag{47}$$

$$BA \to BAB, \qquad BA \to ABA, \tag{48}$$

annihilation of like pairs

$$AA \to \emptyset, \qquad BB \to \emptyset\,, \tag{49}$$

and explicit diffusion of A-s and B-s. The space-time evolution of this system looks very similar to that of the PCPD model (Fig.7), but now the compact domains are built up from alternating sequences of A-s and B-s. Again the critical exponents determined by simulations agree with those of the PCPD model. Note that hard-core particle exclusions

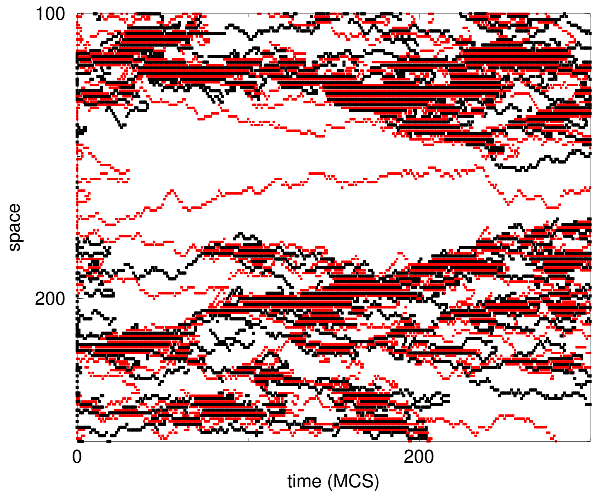

FIGURE 7. Space-time evolution of the two component PCPD model.

may play an important role in 1d multi-component models [59] and no such transition was found in models where the particle creations are replaced by

$$AA \to AAB, \quad BB \to ABB, \quad \text{or} \quad AB \to ABB \ \ldots \quad \text{etc.} \tag{50}$$

processes.

2+1d binary spreading model results

While the bosonic field theory of binary production models predicted $d_c = 2$ upper critical dimension [48], preliminary field theoretical investigations of fermionic particles suggested $d_c = 1$ [60]. Since the bosonic field theory was shown to be valid in the inactive phase only the d_c of lattice models with site restrictions had been an open question for some time. A 2+1 dimensional binary spreading model with the dynamics

$$A\emptyset \overset{D}{\leftrightarrow} \emptyset A, \quad 2A \overset{p(1-D)}{\longrightarrow} \emptyset, \quad 2A \overset{(1-p)(1-D)}{\longrightarrow} 4A \tag{51}$$

was investigated in [54]. Note that the parity conservation is expected to be irrelevant. This was shown in 1d by simulations [52]. The density decay simulations in $L = 200-2000$ systems showed that that the particle density $\rho(t)$ and the pair density $\rho_2(t)$ scale in the same way except in the inactive phase. Numerical data supports mean-field scaling with logarithmic corrections of the form (38) (see Fig.8). Similarly the steady state data exhibits logarithmic corrections of the form (39) that is the smallest at $D = 0.5$ (see Fig.9). These results are summarized in Table 2. The mean-field scaling has also been confirmed by finite size scaling studies of the system.

In the inactive phase the $A + A \to \emptyset$ reaction governs the particle density decay. This process was solved exactly by Lee [9] who predicted the following late time scaling

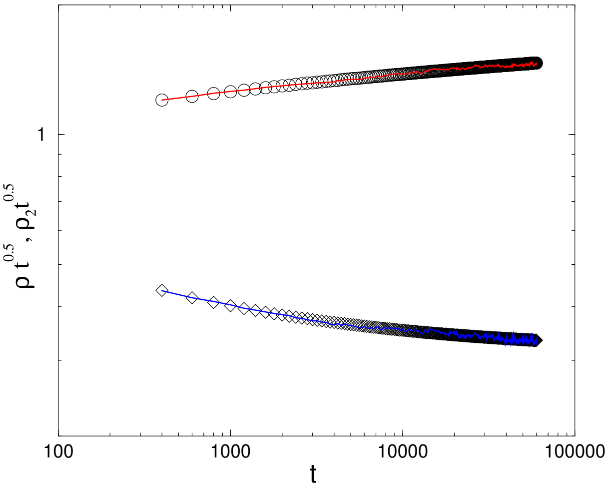

FIGURE 8. Logarithmic fit (circles) for $\rho(t)$ (upper curve) and with the form (39) (diamonds) for $\rho_2(t)$ (lower curve) at $D = 0.2$ and $p = 0.41235$.

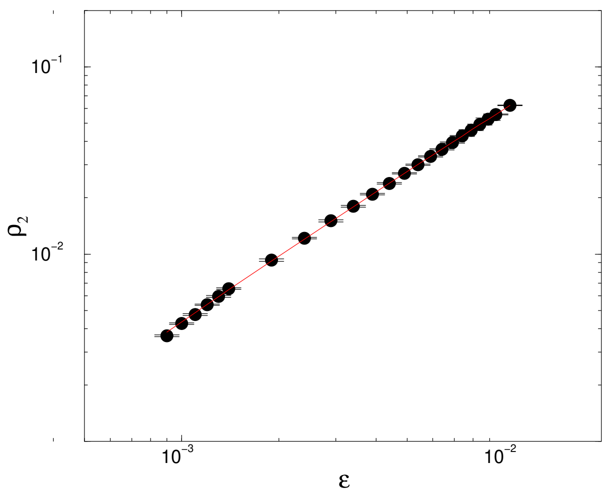

FIGURE 9. Logarithmic fitting to $\rho_2(\infty)$ at $D = 0.5$ using the form eq.(39). The coefficients are $a = 0.112$, $b = 0.01$ and $\beta = 0.96(5)$.

TABLE 2. Summary of simulation results of the 2d binary production model at criticality.

	$D = 0.2$	$D = 0.5$	$D = 0.8$
p_c	0.4124(1)	0.4394(1)	0.4751(1)
α	0.507(10)	0.496(6)	0.497(10)
α_2	0.501(10)	0.501(5)	0.484(15)
β	1.07(10)	1.01(10)	1.07(10)
β_2	1.03(8)	0.96(5)	0.95(5)

behavior in $d = 2$

$$\rho(t) = \frac{1}{8\pi D} \ln t/t + O(1/t) \tag{52}$$

The simulations showed that initially the density decays faster than this power-law but later crosses over to the expected eq.(52) behavior with amplitude $0.078(2)$ and with a $4.46/t$ correction to scaling term. On the other hand the pair density becomes decoupled (unlike at criticality) and decays as $\rho_2 \propto t^{-2}$ in agreement with the mean-field prediction.

SUMMARY AND OUTLOOK

In this paper I introduced the most well known reaction-diffusion models exhibiting nonequilibrium phase transitions to absorbing states via field theoretical formalism. I showed the symmetries and conservation laws that are responsible for different universality classes of the critical phase transitions. I also gave a brief introduction to some multi-component systems, where a slave field is coupled to a DP process.

Binary production systems introduced about two year ago exhibit robust, novel type of critical behavior. I showed that numerical data suggest that the upper critical dimension of fermionic lattice models is $d_c = 2$ in agreement with bosonic field theoretical predictions. Hence non-trivial critical behavior appears in 1d only. However bosonic field theory fails to describe the behavior of this class in 1d and fermionic field theory has not been successful yet. The action of this model does not exhibit any special symmetries or conservation laws that makes it exceptionally from the classes mentioned in the first part. Also the parity conservation seems to be irrelevant for such models. It would be interesting to check the effect of other conservation laws as well. The effect of diffusion strength has also been shown through the results of recent high precision simulations and cluster-mean-field approximations. It has also been proven that such binary production models can be considered as coupled two-component systems that may provide a clue for further analytical approaches.

Very recently models with more general $nA \to (n+m)A$ type of particle production ($n > 2$) and explicit diffusion have been introduced and novel kind of critical behavior is reported by simulations [43, 44, 45]. These models exhibit similar spatio-temporal evolution as the binary production ones, and are subject of current research.

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