Universal scaling behavior in nonequilibrium system

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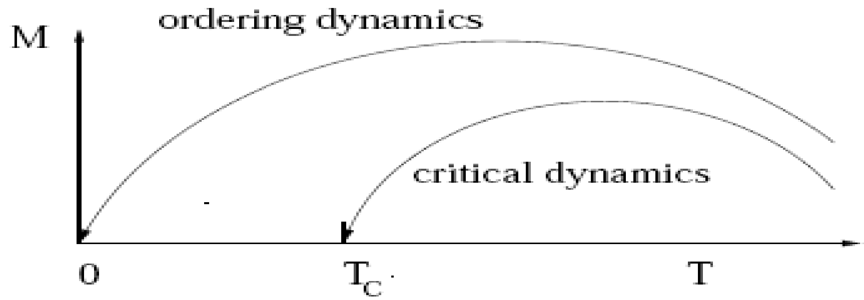
- Nonequilibrium system occur when:
  a) Relaxation to thermal equilibrium starting far from equilibrium



$$L(t) \sim \xi(t) \sim t^{1/Z} ,$$

  b) By applying external fields or forcing currents fluxes or connecting to different local thermal bathes
  c) System defined purely by transtion rates (reaction/diffusion model, surface growth system, spin system ...

- For b) and c): Broken detailed balance in general:

$$w_{i \to j} P(s_i) = w_{j \to i} P(s_j)$$

$\Rightarrow$ No Gibbs distribution, no free energy potentials

Scaling in nature

Scaling (rescaling invariance): $r \rightarrow br, \quad F(r,t,...) \rightarrow a^x F(br, b^z t, ...)$

behavior is frequent in nature:

Geometrical scaling (models), fractal objects

Photograph of a romanesco broccoli, showing a naturally occuring fractal

Snowflake viewed in an optical microscope

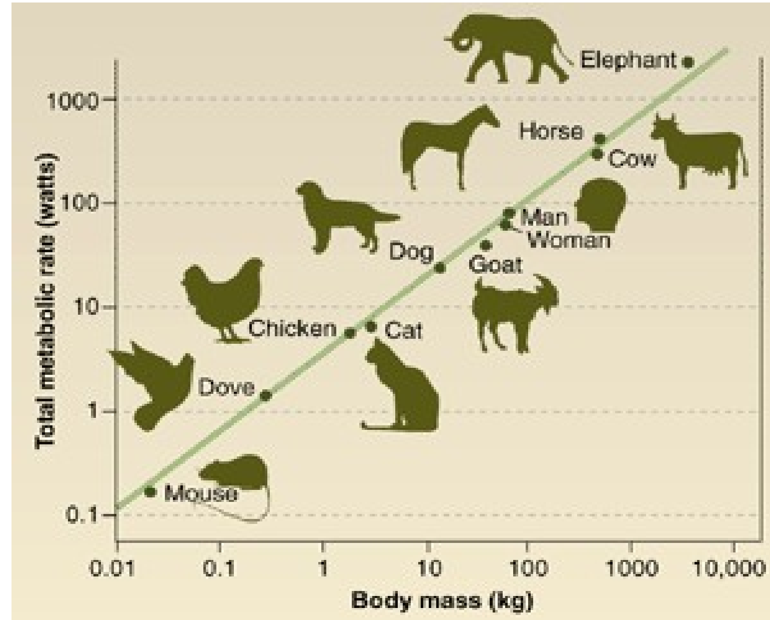
Rescaling invariance results in power-laws (at least for some decades)

(+ log. periodic oscillations for b discrete), diverging fluctuations

Power-law scaling in nature

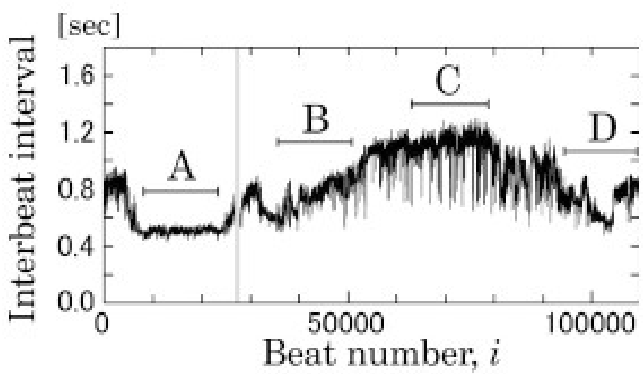
Nontrival scaling in biological system

- Life-span tends to lengthen--and metabolism slows down--in proportion to the quarter power of an animal's body weight.



- Biological control systems :

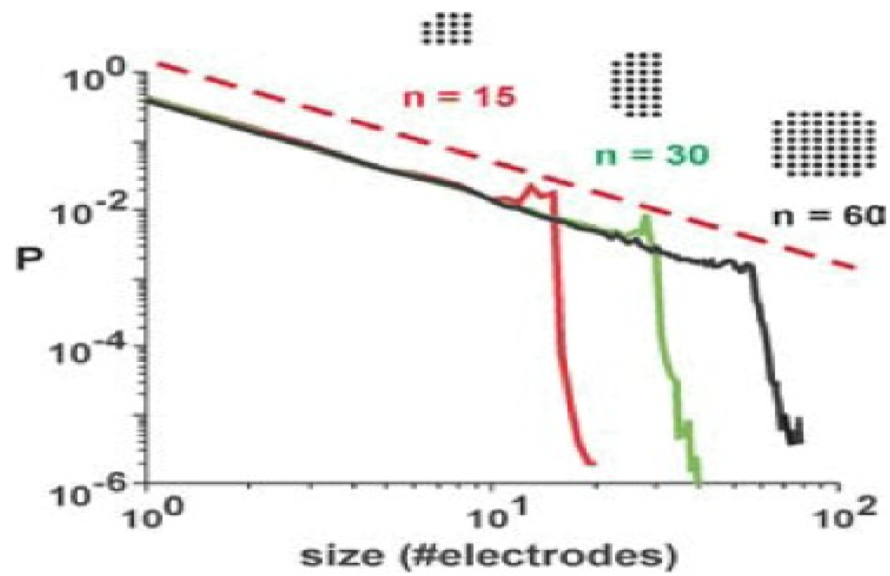
K. Kiyono, et al., PRL95 (2005) 058101.



critical, diverging fluctuations and phase transitions at different periods of activity

- Brain :

The size distribution of neuronal avalanches

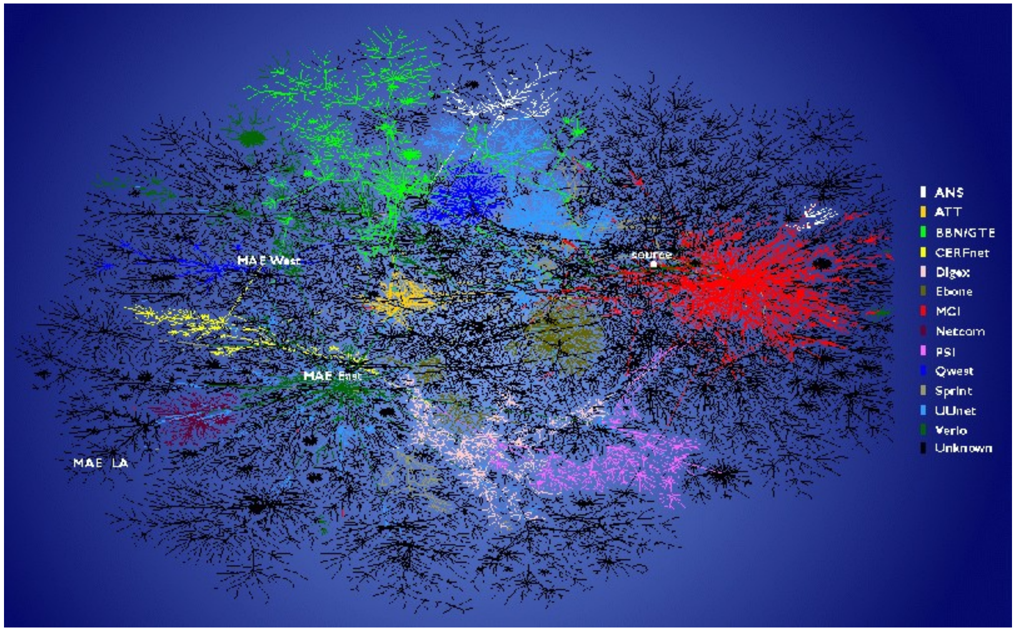


G. Werner : Biosystems, 90 (2007) 496,

Power-law scaling in nature

Man-made system:

Internet, WWW, ... etc.



- Self similarity in internet networks
- Heavy-tailed file size distribution in web traffic
- Heavy-tailed distributed on/off processes on TCP

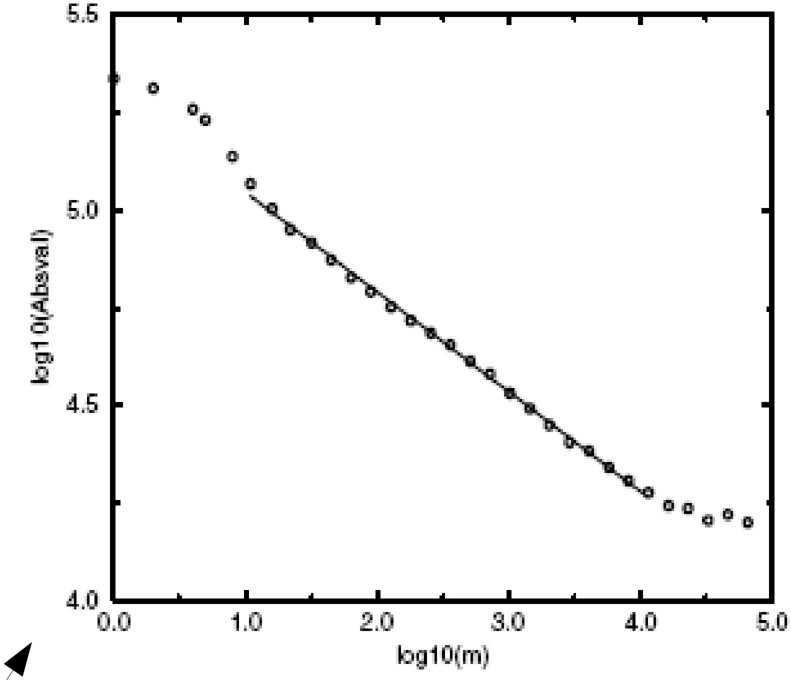
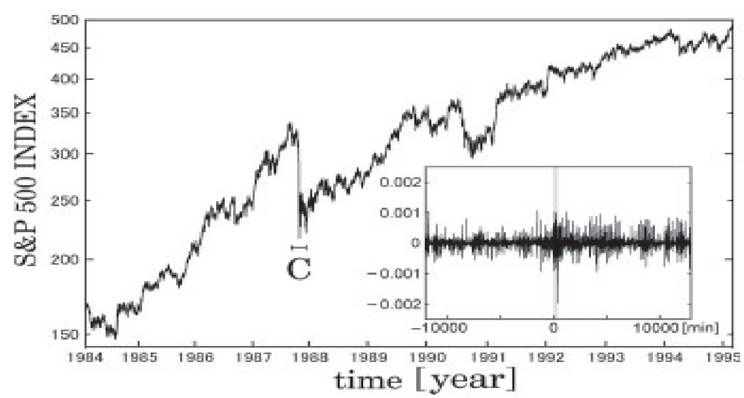


Figure 1: Scaling analysis of the traffic generated by a file transfer logged at the client side. a) Absolute

Stock-prize fluctuations and markets:

K. Kiyono, et al., PRL96 (2006) 068701

Log-periodic divergence of fluctuations near market crashes

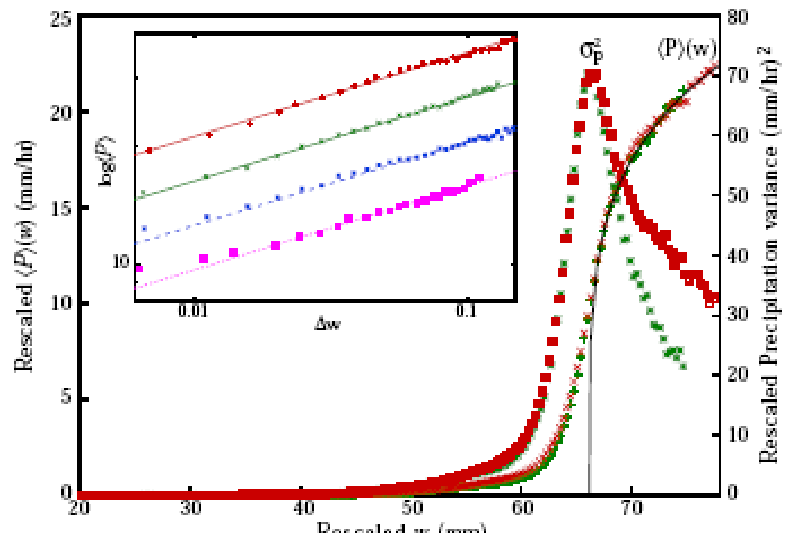


Power-law scaling in nature

Meteorology and Climatology:

O.Petres and D. Neelin, Nature Phys. 2 (2006)393

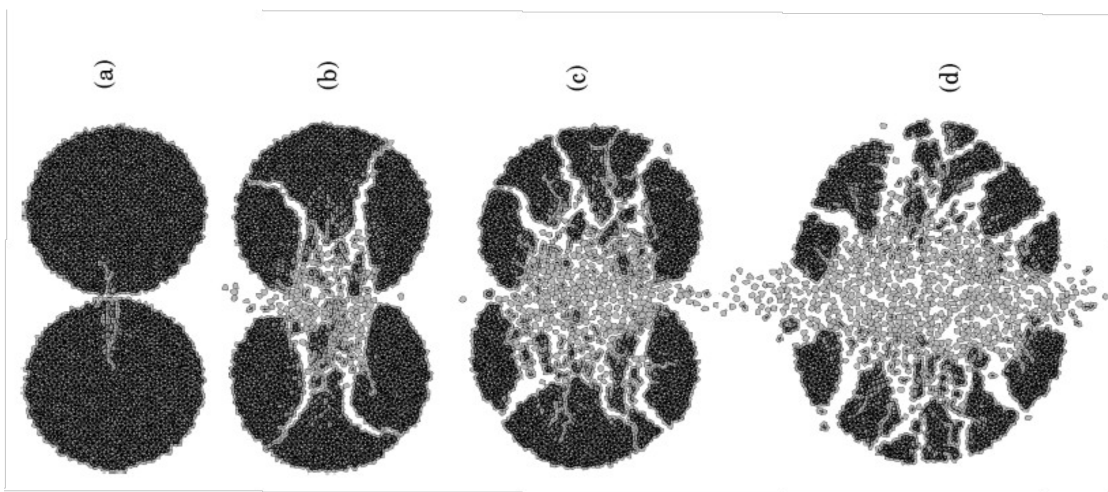
rain fall distribution

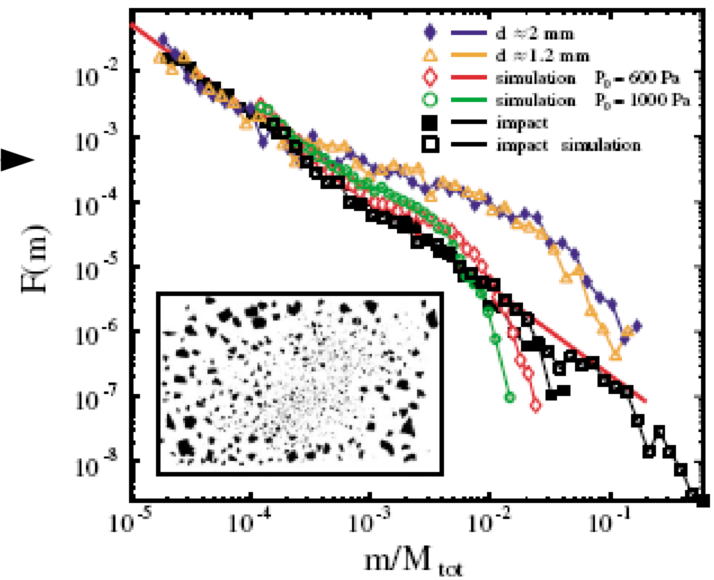


Earthquake size distribution

Damage formation by collosions, explosions

(*F. Kun & H. Hermann)*





And many more

What is the origin ?

Overview of equilibrium (static) critical systems

- Power-laws occur at second-order (continuous) phase transitions
- At criticality, when ξ (correlation length) diverges the critical exponents:

$$c_H \propto \alpha_H^{-1}\left((|T-T_c|/T_c)^{-\alpha_H}-1\right) \;, \tag{1.1}$$

$$m \propto (T_c - T)^{\beta} \;, \tag{1.2}$$

$$\chi \propto |T-T_c|^{-\gamma} \;, \tag{1.3}$$

$$m \propto H^{1/\delta_H} \;, \tag{1.4}$$

$$G_c^{(2)}(r) \propto r^{2-d-\eta_\perp} \;, \tag{1.5}$$

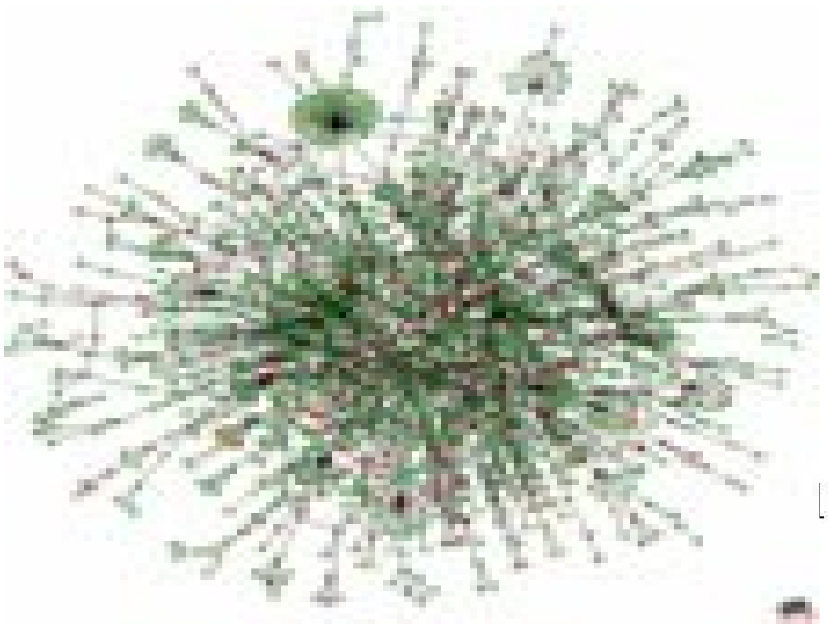
$$\xi \propto |T-T_c|^{-\nu_\perp} \;. \tag{1.6}$$

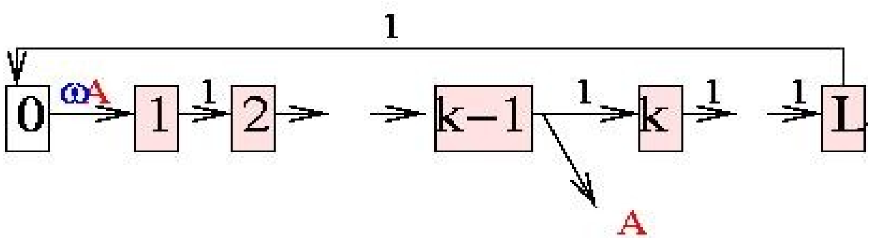
- They are related by scaling laws:

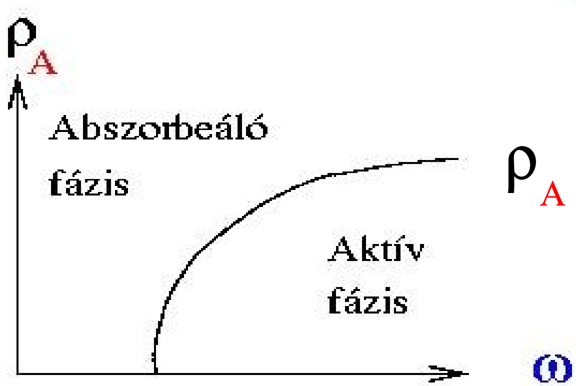
$$\alpha_H + 2\beta + \gamma = 2,\;\; \alpha_H + \beta(\delta_H + 1) = 2, \tag{1.7}$$
$$(2-\eta_\perp)\nu_\perp = \gamma_H,\;\; \nu_\perp d = 2 - \alpha_H \;.$$

- Sets of exponents define **Universality classes**

Generalization to nonequilibrium







$$\rho_A \propto (\omega - \omega_c)^{\beta}$$

H. Berry, Phys. Rev. E 2003

- Enzymological exmaple:

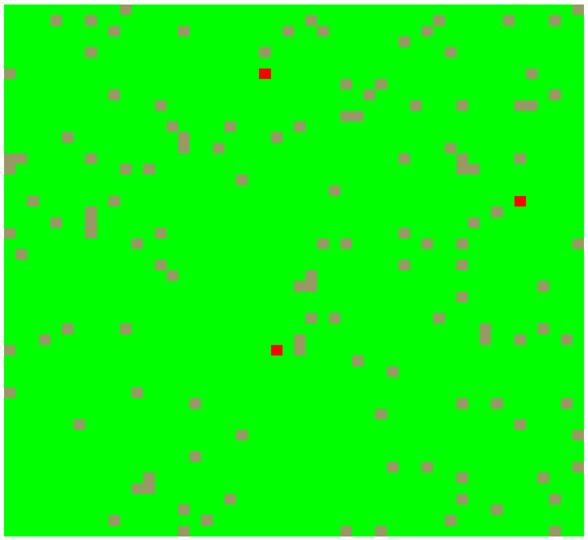
  Continous phase transition with $\xi \to \infty$ between **active** and **absorbing** states

  Critical behavior with power-laws and scaling relations

  Scaling near ω_c

Nonequilibrium critical phase transitions appear in models of

- Epidemics spreading : *T. Ligget, Interacting particle systems 1985*



The *ABC* model:

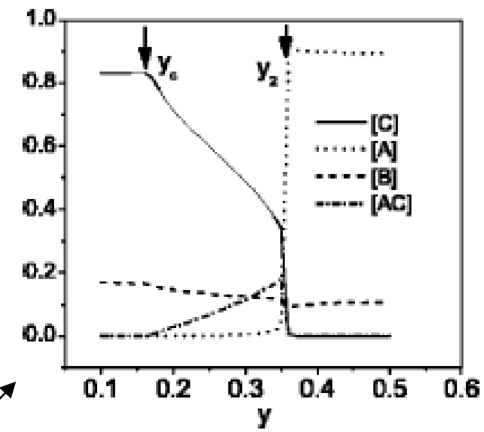
(a) $A_g + * \rightarrow A_{ads}$,

(b) $A_{ads} \rightarrow A_g$,

(c) $BC_g + 2* \rightarrow B_{ads} + C_{ads}$,

(d) $A_{ads} + C_{ads} \rightarrow AC_g + 2*$,

(e) $B_{ads} + B_{ads} \rightarrow B_{2(g)} + 2*$.



- Catalysis : *Da-yin Hua, Phys. Rev E 70 (2004) 066101*,
- Itinerant electron systems : *D. E. Feldman, Phys. Rev. Lett 95 (2005) 177201*,
- Cooperative transport : *S. Havlin and D. ben-Avraham, Adv. Phys. 36 (1987) 695*

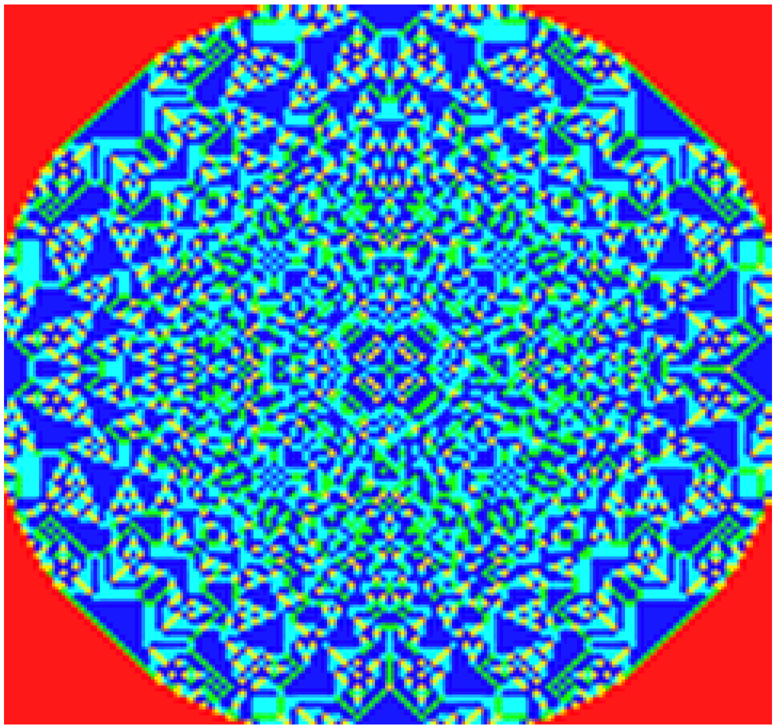
… and many more (a whole zoo of models)

Universality classes ?

Do we need tuning ?

Scaling laws in “self-organized critical systems” (SOC)

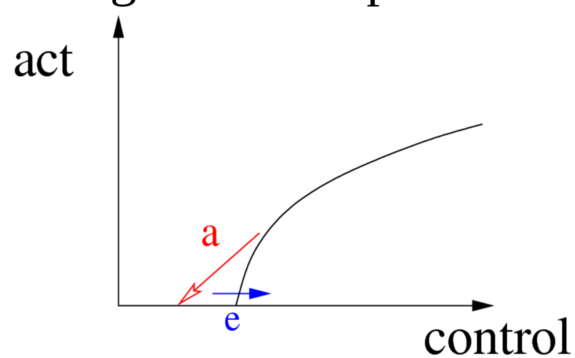
Bak-Tang-Wiesenfeld sandpile model



40000 grains dropped on center of 120 x 120 lattice with h_c=4

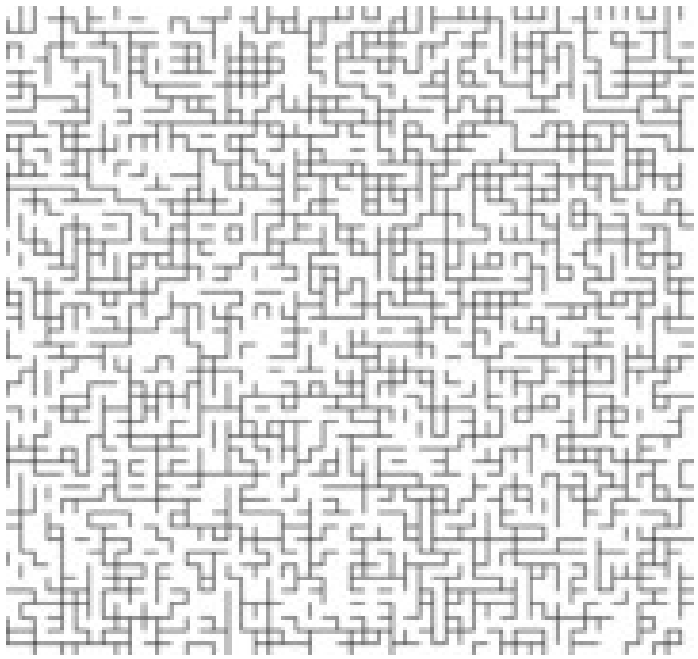
Add a grain of sand:
$h(x,y) \rightarrow h(x,y) + 1$
And avalanche if: $h(x,y) > h_c$:
$h(x, y) \rightarrow h(x, y) - 4$
$h(x \pm 1, y) \rightarrow h(x \pm 1, y) + 1$
$h(x, y \pm 1) \rightarrow h(x, y \pm 1) + 1$

- SOC mechanism has been proposed to model earthquakes, the evolution of biological systems, solar flare occurrence, fluctuations in confined plasma, snow avalanches and rain fall for example...
- The term SOC usually refers to a mechanism
  Slow energy accumulation (e)
  Fast energy redistribution (a)
  driving a system towards a critical state.
  Prototype: sandpile model
- Self-tuning to critical point:



- SOC models can be mapped onto ordinary nonequilibrium criticality of phase transition to absorbing phase

Static, random percolation



Detail of a bond percolation on the square lattice in two dimensions with percolation probability p=51

Besides random percolation correlated percolation exists. This can occur at thermal phase transition points: Fortuin-Kastaleyn construction.

- Ordinary(static) percolation (*see Stauffer & Aharony (1994)).*
- Sites or bonds placed randomly on lattices, above $p > p_p$ infinitely large connected cluster.
  Different cluster definitions: $b = 1 - \exp(-2J/kT)$.
  Diverging correlation length:

$$\xi(p) \propto |p - p_p|^{-\nu_\perp} \quad ,$$

- Critical quantities:
  cluster size: $n_s \propto s^{-\tau} f(|p - p_p| s^{\sigma})$.
  moments:

$$\sum_s n_s(p) \propto |p - p_p|^{2-\alpha_p} \; ,$$

$$\rho_\infty = \sum_s s n_s(p) \propto |p - p_p|^{\beta} \; ,$$

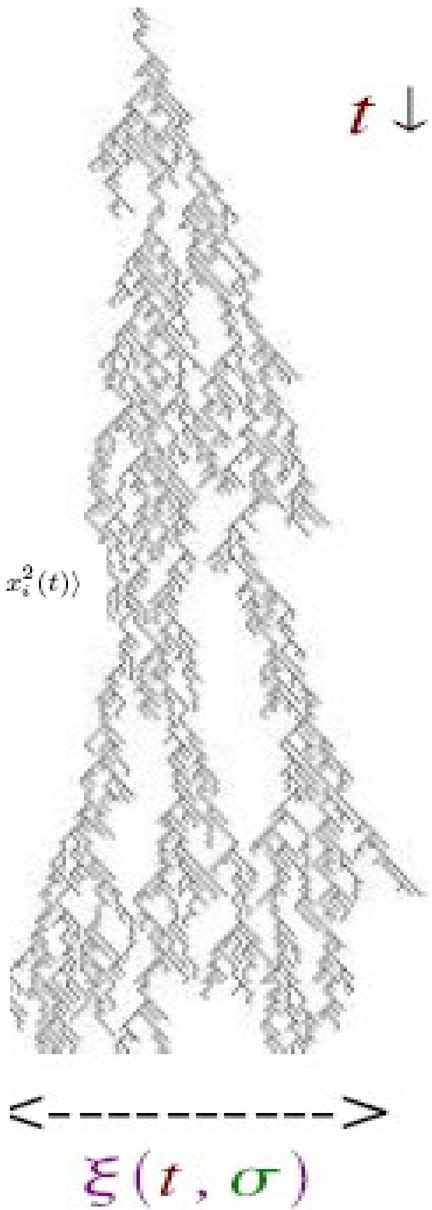
$$<\rho_\infty>^2 - <\rho_\infty{}^2> = \sum_s s^2 n_s(p) \propto |p - p_p|^{-\gamma} \; ,$$

$$\sum_s s n_s(p) e^{-hs} \propto h^{1/\delta_p} \quad .$$

- Scaling laws:

$$\tau = 2 + 1/\delta_p, \quad 1/\sigma = \beta\delta_p$$

Percolation dynamics



$$R^2(t) = \frac{1}{N(t)} \langle \sum_i x_i^2(t) \rangle$$

- Order parameter 1 : density of active sites

$$\rho(t) = \frac{1}{L^d} \langle \sum_i s_i(t) \rangle ,$$

which in the supercritical phase vanishes with the leading power behavior

$$\rho^{\infty} \propto |p - p_c|^{\beta} , \qquad (1.35)$$

- Order parameter 2 (dual) final survival probability

$$P_{\infty} \propto |p - p_c|^{\beta'}$$

- Critical dynamical behvior at: $\Delta = |p\text{-}p_c|$

$$\tau \propto \Delta^{-\nu_{\|}} \qquad \xi_{\perp} \propto \Delta^{-\nu_{\perp}}$$

The critical long-time behavior of these quantities are described by

$$\rho(t) \propto t^{-\alpha} f(\Delta\, t^{1/\nu_{\|}}) , \qquad P(t) \propto t^{-\delta} g(\Delta\, t^{1/\nu_{\|}}) , \qquad (1.37)$$

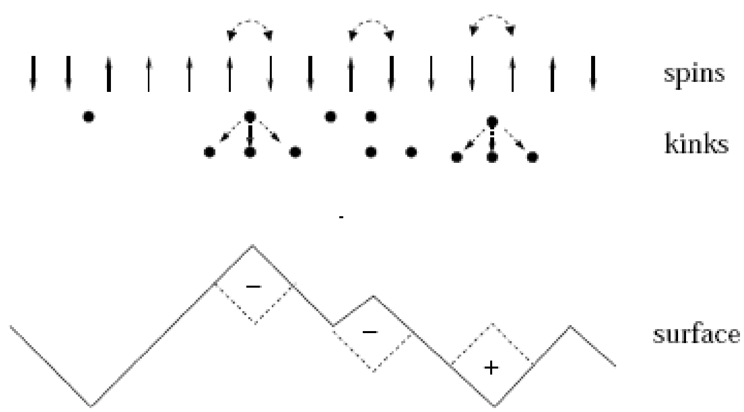
where α and δ are the critical exponents for decay and survival, $\Delta = |p - p_c|$,

- For short times:

$$R^2(t) \propto t^z \qquad N(t) \propto t^{\eta}$$

Genuine basic models with absorbing state

- Detailed balance cond: $$w_{i \to j} P(s_i) = w_{j \to i} P(s_j)$$ is broken
- Defined by the transition rates: $w_{i \to j}$
- Field theory based on non-Hermition „Hamiltonian"
- The ordered state exhibits small fluctuations: no return in case of falling in it (no $0 \to A$ reaction)
- Phase transitions are possible in low *(1,2)* dimenions. MW theorem is not valid!
- Reaction-diffusion particle systems : Competition of creation, removal, diffusion
- Usually bosonic field theory is applied but in low dimensions topological constraints (particle exclusion) becomes relevant
- Mapping onto spin and surface models is possible:



Mean-field classes of site restricted, one-component reaction-diffusion systems *(fermionic)*

- General, reaction-diffusion systems :

Order parameter ρ

$$nA \xrightarrow{\sigma} (n+k)A \qquad mA \xrightarrow{\lambda} (m-l)A \qquad 0A \overset{D}{\Leftrightarrow} A0$$

$$\rho(t) \propto t^{-\alpha}$$

$$\rho \propto |\sigma - \sigma_c|^{\beta}$$

- *Assume:* $\lambda = 1 - \sigma$, $\frac{\partial \rho}{\partial t} = K(k)\sigma\rho^n(1-\rho)^k - L(l)(1-\sigma)\rho^m$.
- Inactive phase: $mA \rightarrow (m-l)A$ is dominant: $\rho \propto t^{1/(m-1)}$
- The solution splits into 3 cases: **1.** $n = m$:

The steady state solution in this case can be obtained by solving

$$K(k)\sigma(1-\rho)^k = L(l)(1-\sigma), \tag{4.13}$$

where the trivial ($\rho = 0$) solution (corresponding to the absorbing state) has been factored out. For the active phase one gets

$$\rho = 1 - \left[\frac{L}{K}\frac{1-\sigma}{\sigma}\right]^{1/k}, \tag{4.14}$$

which vanishes

$$\sigma_c = \frac{L}{K+l} > 0 \tag{4.15}$$

with the leading order singularity

$$\rho \propto |\sigma - \sigma_c|^{\beta^{MF}}, \tag{4.16}$$

and order parameter exponent exponent

$$\beta_{MF} = 1\ . \tag{4.17}$$

At the critical point the time dependent behavior is described by

$$\frac{\partial \rho}{\partial t} = -2K^2\rho^{n+1} + O(\rho^{n+2})\ , \tag{4.18}$$

which gives a leading order power-law solution

$$\rho \propto t^{-1/n} \tag{4.19}$$

hence the corresponding mean-field exponents are

$$\alpha_{MF} = \beta/\nu_{||} = 1/n\ . \tag{4.20}$$

Mean-field classes of site restricted, one-component reaction-diffusion systems *(fermionic)*

- **2.** $n < m$:

By factoring out the trivial $\rho = 0$ solution we are faced with the general condition for a steady state

$$K\sigma(1-\rho)^k = L(1-\sigma)\rho^{m-n} . \qquad (4.22)$$

One can easily check that in this case the critical point is at

$$\sigma_c = 0 \; , \qquad (4.23)$$

(see Fig. 4.1(b)) and the density decays as

$$\alpha = 1/(m-1) \; , \qquad (4.24)$$

as for branching ($n = 1$) and $m = l$ particle annihilating models (BmARW) showed by Cardy and Täuber [Cardy and Täuber (1996)]. However the steady state solution for $n > 1$ gives different β exponents than those of BmARW classes, namely

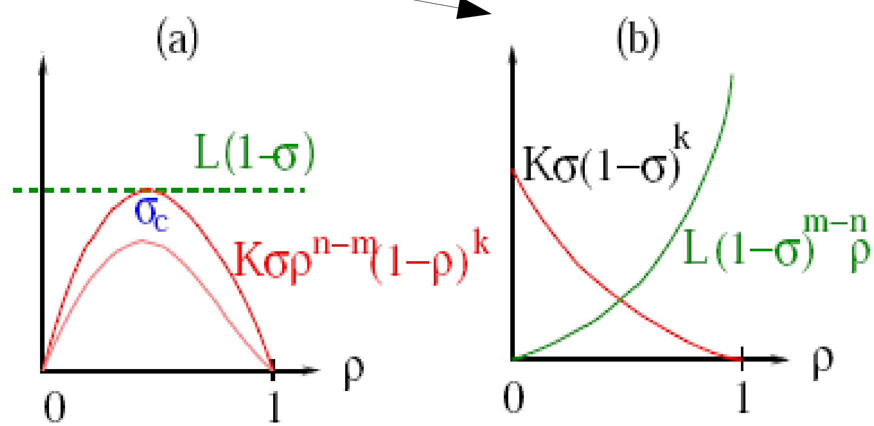
$$\beta = 1/(m-n) \; . \qquad (4.25)$$

- **3.** $n > m$:

In this case besides the $\rho = 0$ absorbing state solution we can get an active state if

$$K\sigma\rho^{n-m}(1-\rho)^k = L(1-\sigma) \qquad (4.21)$$

is satisfied. Both sides are linear functions of σ such that for $\sigma \to 0$ only the $\rho = 0$ is a solution. The left hand side is a convex function of ρ (from above) with zeros at $\rho = 0$ and $\rho = 1$. Therefore by increasing σ from zero the left hand side meets the right hand side at σ_c, $\rho_c > 0$ (see Fig. 4.1(a)). If this solution is stable, a discontinuous transition of the steady state density (order parameter) takes place by varying the reaction rates.



***n* and *m* determine the (site) mean-field class!**
Diffusion does not play a role

G. Ódor: PRE 67, 056114 (2003)

– ***What about fluctuation effects below d_c ?***

Upper critical dimension and below. Numerical example

G. Ódor, Phys. Rev. E 73, 047103 (2006)

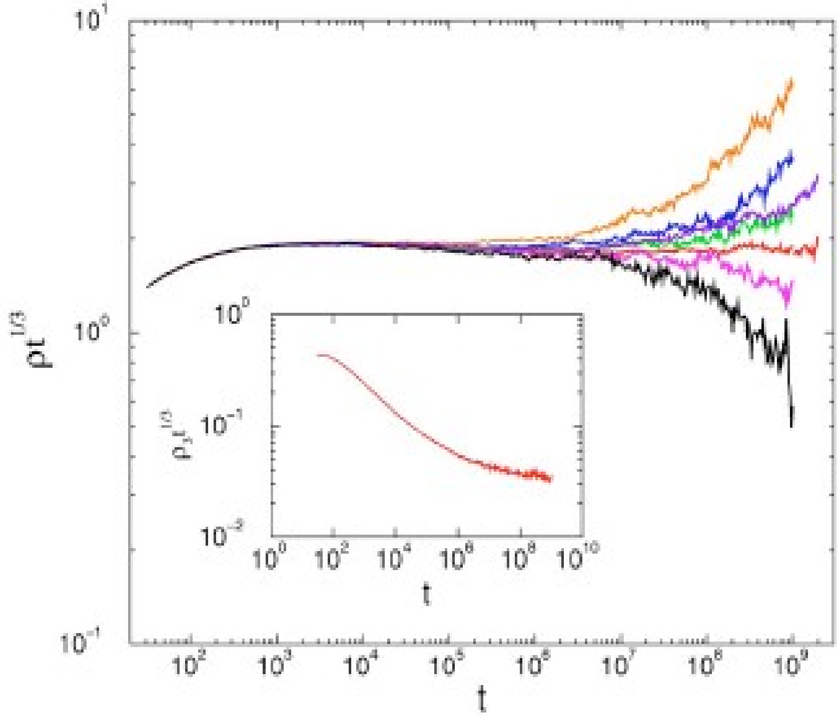


FIG. 4. (Color online) Density decay in the $3A \to 4A$, $3A \to \emptyset$ model at $D=0.8$. Different curves correspond to $p=0.1185$, 0.1187, 0.118 75, 0.1188, 0.118 86, 0.1189, and 0.119 (from top to bottom). The inset shows the decay of triplets for $p=0.11886$.

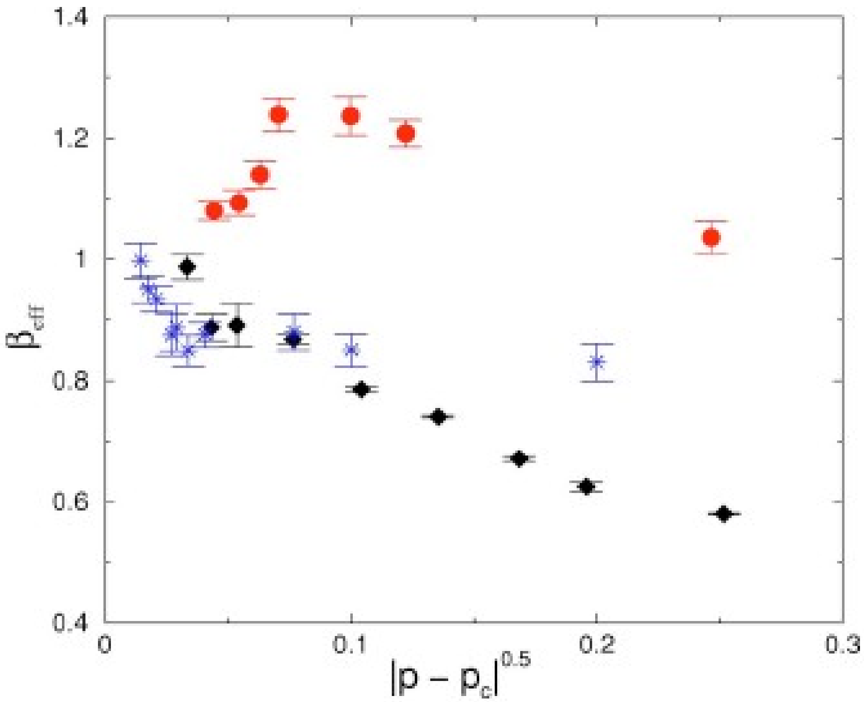


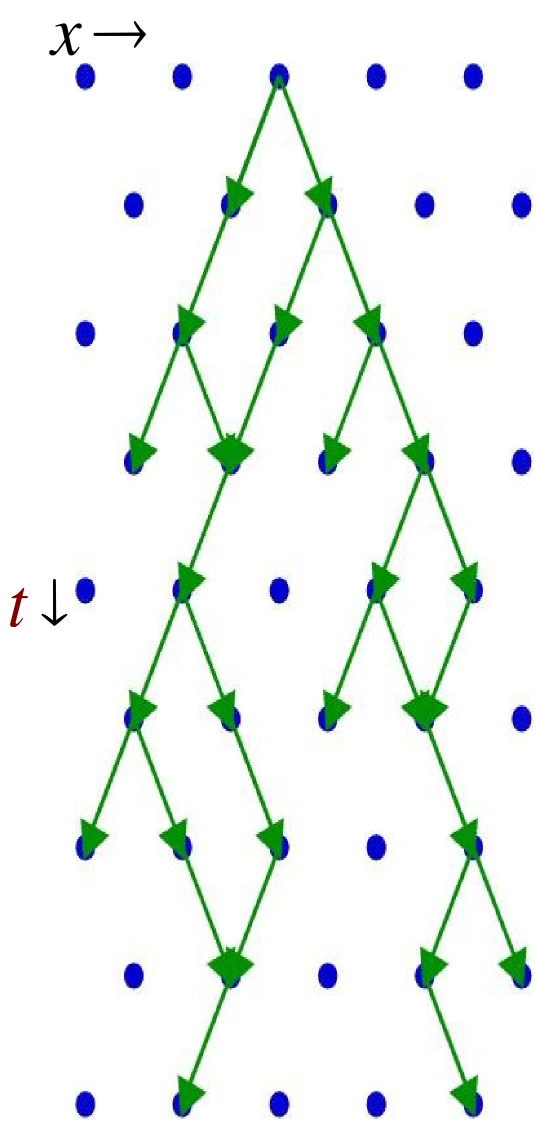
FIG. 3. (Color online) Effective static order parameter exponent results in the active phase. Bullets correspond to $D=0.8$, boxes to $D=0.1$ of the $3A \to 4A$, $3A \to 2A$ model. Stars denote the results of the $3A \to 4A$, $3A \to \emptyset$ model.

$$\beta_{eff}(p_i) = \frac{\ln \rho(\infty, p_i) - \ln \rho(\infty, p_{i-1})}{\ln(p_i) - \ln(p_{i-1})},$$

$$\beta_{eff} = \frac{d \ln[\rho(\infty)]}{d \ln(\epsilon)} = \beta + \frac{x}{\ln(\epsilon)}$$

Branching and annihilating random walks (BARW)

Unari reaction models



$$A \xrightarrow{\sigma} (n+1)\,A,\;\; A \rightarrow 0,\;\; 2A \rightarrow A$$

$\Leftarrow$ *1+1 d* realization $\Rightarrow$

For *n=1* directed percolation (DP), contact process, epidemic spreading :

In the reaction-diffusion language the DP is built up from the following processes

$$A \xrightarrow{\gamma} \emptyset \quad A\emptyset \overset{D}{\leftrightarrow} \emptyset A \quad A \xrightarrow{\sigma} 2A \quad 2A \xrightarrow{\lambda} A\,. \tag{4.31}$$

The mean-field equation for the coarse-grained particle density $\rho(t)$ is

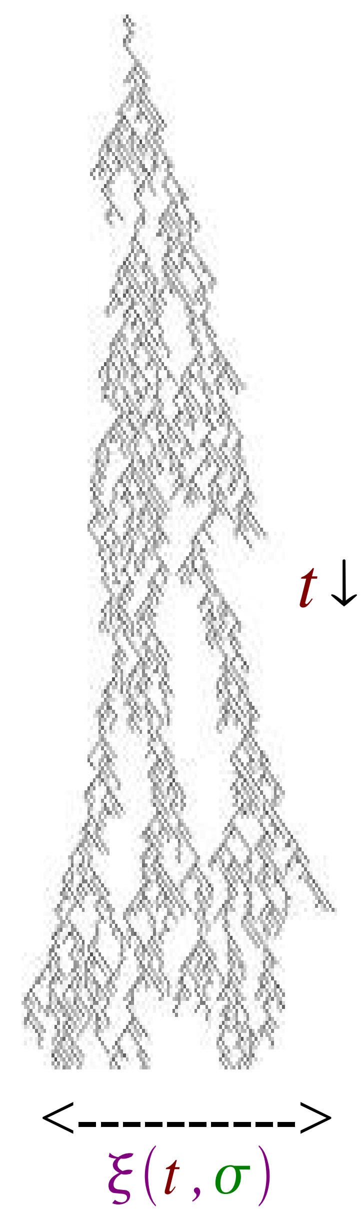
$$\frac{d\rho}{dt} = (\sigma - \gamma)\rho - (\lambda + \sigma)\rho^2 \;\; . \tag{4.32}$$

This has the stationary stable solution

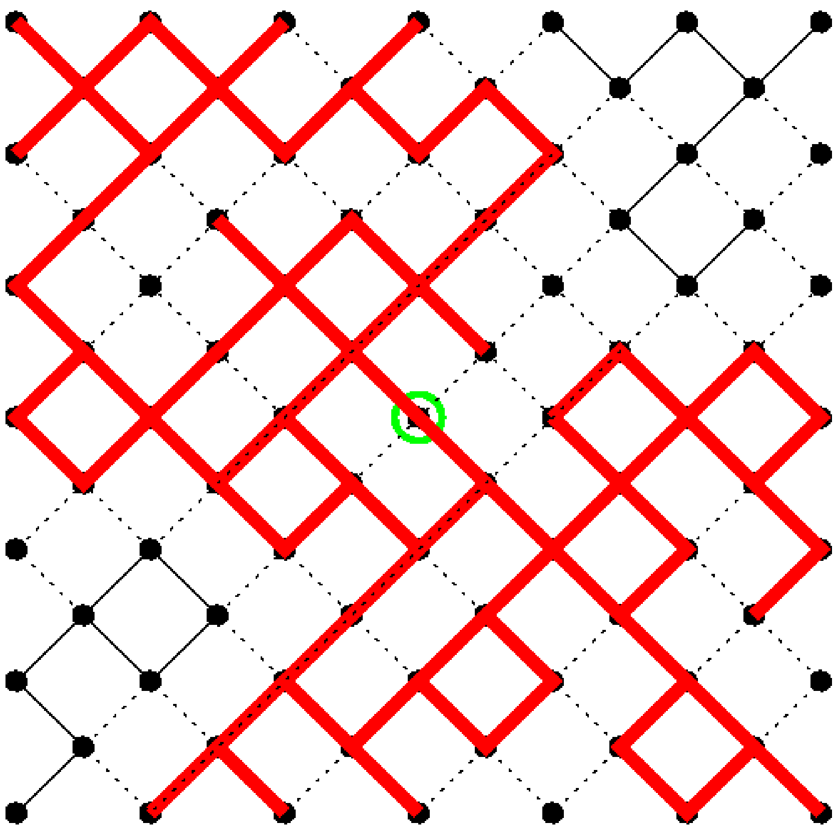
$$\rho(\infty) = \begin{cases} \frac{\sigma-\gamma}{\lambda+\sigma} & \text{for} \;:\; \sigma > \gamma \\ 0 & \text{for} \;:\; \sigma \le \gamma \end{cases} \tag{4.33}$$

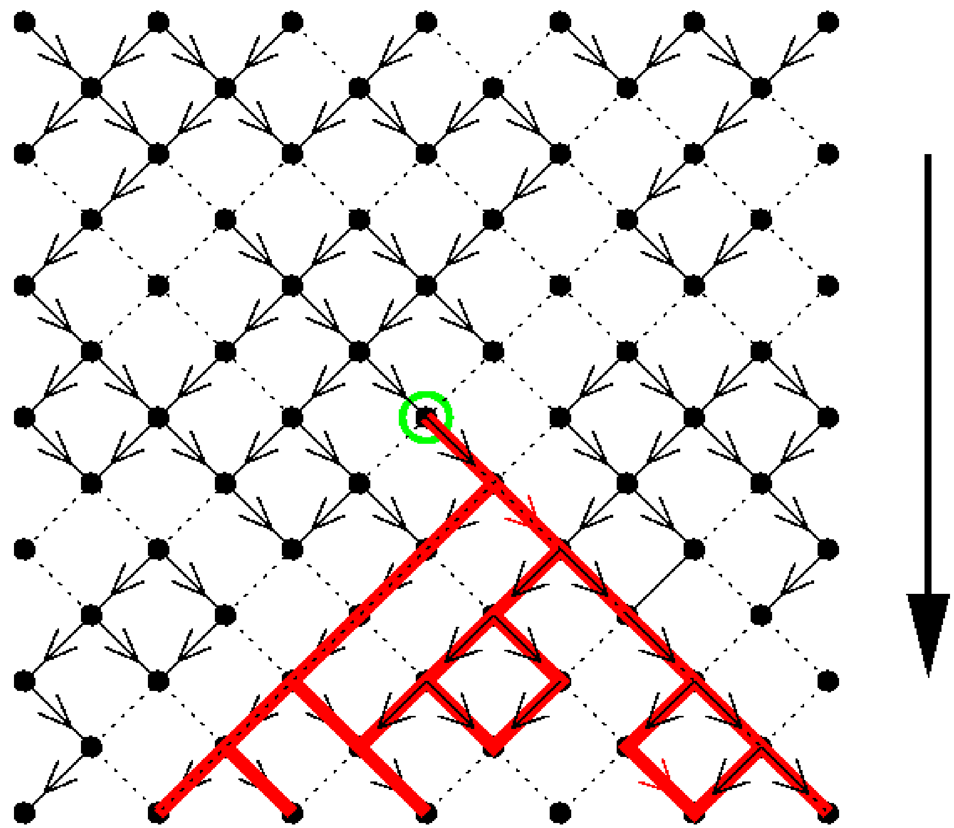
exhibiting a continuous transition at $\sigma = \gamma$. A small variation of σ or γ near the critical point implies a linear change of ρ. Therefore the order parameter exponent in the mean-field approximations

$$\beta_{MF} = 1\;. \tag{4.34}$$



2 dim IP versus 1+1 dim DP





DP cluster (generalized) mean-field (GMF)

- Master equation for n-point configuration probabilities of s_i

$$\frac{\partial P_n(\{s_i\})}{\partial t} = f(P_n(\{s_i\})), \qquad (1)$$

- Bayesian extension process ($n>N$ correlations are neglected)

$$P_n(s_1, \dots, s_n) = \frac{\prod_{j=0}^{j=n-N} P_N(s_{1+j}, \dots, s_{N+j})}{\prod_{j=1}^{j=n-N} P_{N-1}(s_{1+j}, \dots, s_{N-1+j})}. \qquad (2)$$

- Reduction of parameters due to symmetries, conservations

$$P_n(s_1, \dots, s_n) = \sum_{s_{n+1}} P_{n+1}(s_1, \dots, s_n, s_{n+1}),$$

$$P_n(s_1, \dots, s_n) = \sum_{s_0} P_{n+1}(s_0, s_1, \dots, s_n).$$

- If we apply GMF for one-dimensional, site restricted lattice version of DP, for $N=10$ we have *528* independent variables

DP cluster generalized mean-field (GMF)

G. Szabó and G. Ódor PRE49 (1994) 2764, G. Ódor PRE51 (1995) 6261

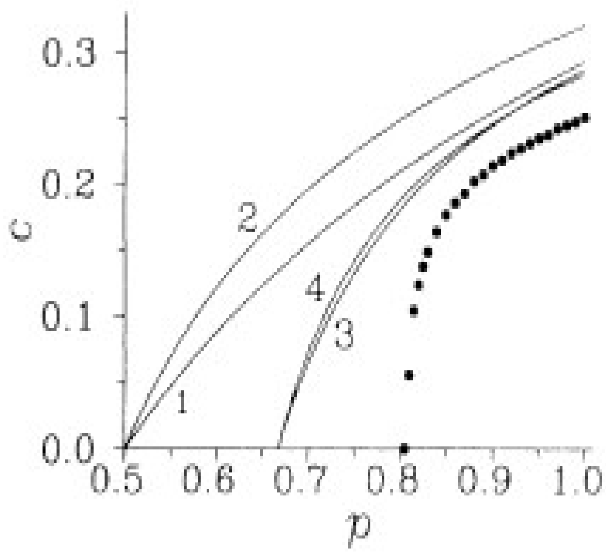


FIG. 1. Average concentration vs p suggested by n-point approximations (labels refer to n).

Rule 6/16 SCA to realize DP in 1d

$t-1$:	0 0	0 1	1 0	1 1
t:	0	1	1	0

creation with prob. p

GMF + Coherent anomaly (CAM) extrapoltion

$$Q_n \sim a(n)(p/p_c^n - 1)^{\omega_{cl}},$$

$$a(n) \sim (p_c^n - p_c)^{\omega - \omega_{cl}},$$

N=1 and *N=2* exact results:

$$c = \frac{1}{4}\left(2 - \frac{1}{p}\right) \qquad c = \frac{2-3p}{4-8p}$$

In higher order numerical solution
+ Taylor series, Padé extrapolation

$$c = \frac{1}{4}\sum_{k=0}^{\infty} a_k (1-p)^k$$

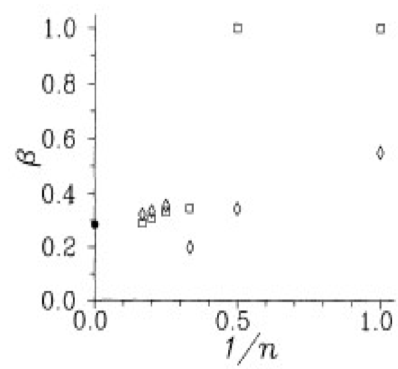
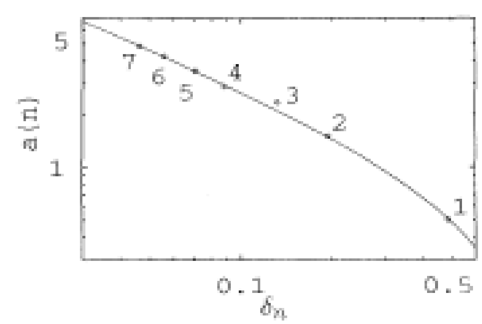


FIG. 4. Critical exponents vs $1/n$ suggested by the Padé approximants Q_1^1 (◇) and Q_2^2 (□).



Padé extrapolation, Ref. [4]	0.29
Simulation, Ref. [15]	0.285(5)
Series expansion for DP, Ref. [1]	0.2769(2)

DP field theory, numerical results

full field theoretical action with the couplings m, $u = \sigma$, $v = \lambda/2$ looks like

$$S = \int d^d x dt [H_D + H_R] = \int d^d x dt \left[\psi(\partial_t - D\nabla^2)\phi + (m + u\psi - v\phi)\psi\phi\right] \ , \tag{4.40}$$

where ϕ is the density, ψ is the response field (appearing in response functions). The action is invariant under the following rapidity-reversal symmetry

$$\phi(x,t) \to -\psi(x,-t) \ , \ \ \psi(x,t) \to -\phi(x,-t) \ . \tag{4.41}$$

This symmetry yields [Grassberger and de la Torre (1979); Muñoz *et al.* (1997)] [2] the scaling relations

$$\beta = \beta' \tag{4.42}$$

$$4\delta + 2\eta = dz \ . \tag{4.43}$$

This field theory was found to be equivalent [Cardy and Sugar (1980)] to the Reggeon field theory [Abarbanel *et al.* (1975); Brower *et al.* (1978)], which is a model of scattering elementary particles at high energies and low-momentum transfers. The the upper critical dimension of directed percolation is can be deduced from the general form (4.30)

$$d_c = 4 \ . \tag{4.44}$$

Table 4.1 Estimates for the critical exponents of directed percolation. Data are from: [Jensen (1999a)] (1d), [Voigt and Ziff (1997)](2d), [Jensen (1992)](3d), [Bronzan and Dash (1974); Janssen (1981)]($4-\epsilon$).

critical exponent	$d=1$	$d=2$	$d=3$	$d=4-\epsilon$
$\beta = \beta'$	0.276486(8)	0.584(4)	0.81(1)	$1-\epsilon/6-0.01128\,\epsilon^2$
$\nu_\perp$	1.096854(4)	0.734(4)	0.581(5)	$1/2+\epsilon/16+0.02110\,\epsilon^2$
$\nu_\parallel$	1.733847(6)	1.295(6)	1.105(5)	$1+\epsilon/12+0.02238\,\epsilon^2$
$Z = 2/z$	1.580745(10)	1.76(3)	1.90(1)	$2-\epsilon/12-0.02921\,\epsilon^2$
$\delta = \alpha$	0.159464(6)	0.451	0.73	$1-\epsilon/4-0.01283\,\epsilon^2$
η	0.313686(8)	0.230	0.12	$\epsilon/12+0.03751\,\epsilon^2$
$\eta_\perp$	1.504144(19)	1.159(15)	1.783(16)	$2+O(\epsilon)$
$\lambda_R/Z = \lambda_C/Z$	1.9(1)	2.75(10)		$4+O(\epsilon)$
θ_g	1.5(2)			$2-5\epsilon/24$
γ_p	2.277730(5)	1.60	1.25	$1+\epsilon/6+0.06683\,\epsilon^2$

This stochastic process can through standard techniques [Janssen (1976)] be transformed into a Langevin equation formalism. Below the critical dimension the RG analysis of the Langevin equation

$$\frac{\partial\rho(x,t)}{\partial t} = D\nabla^2\rho(x,t) + (\sigma - \gamma)\rho(x,t) - (\lambda + \sigma)\rho^2(x,t) + \sqrt{\rho(x,t)}\eta(x,t) \tag{4.45}$$

is necessary [Janssen (1981)]. Here $\eta(x,t)$ is the Gaussian noise field, defined by the correlations

$$< \eta(x,t) > = 0 \tag{4.46}$$

$$< \eta(x,t)\eta(x',t') > = \Gamma\delta^d(x-x')\delta(t-t') \, . \tag{4.47}$$

The noise term is proportional to $\sqrt{\rho(x,t)}$ ensuring that in the absorbing state ($\rho(x,t) = 0$) it vanishes. The square-root behavior stems from the definition of $\rho(x,t)$ as a coarse-grained density of active sites averaged over some mesoscopic box size.

DP below d_c, topological phase diagram method

$$H_R = \gamma(1-p)\,q + \sigma(p^2-p)\,q + \frac{\lambda}{2}(p-p^2)\,q^2$$
$$= \left(\gamma - \sigma p + \frac{\lambda}{2}\,p\,q\right)(1-p)\,q\,. \qquad (4.38)$$

$A \xrightarrow{\gamma} \emptyset \quad A\emptyset \overset{D}{\leftrightarrow} \emptyset A \quad A \xrightarrow{\sigma} 2A \quad 2A \xrightarrow{\lambda} A$

The phase portrait of this system is depicted in Fig. 4.3. The zero energy

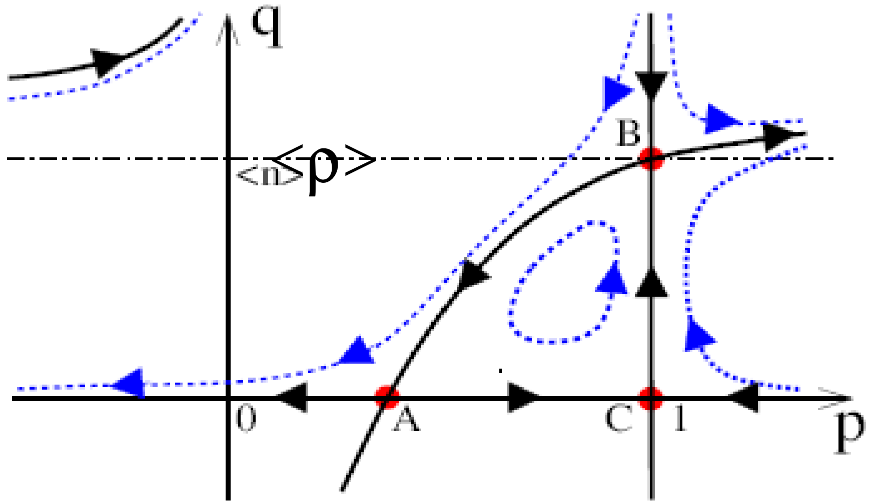


Fig. 4.3 Phase portrait of the DP system in the active phase. Thick solid lines represent zero-energy trajectories, which divide the phase space into a number of disconnected regions. The point $B = \{1, 2(\sigma-\gamma)/\lambda\}$ corresponds to the active mean-field fixed point. From [Elgart and Kamenev (2006)].

lines are the generic $p = 1$ (mean-field) and $q = 0$ (absorbing phase) trajectories, along with the $q = 2(\sigma p - \gamma)/\lambda p$ curve. According to the mean-field analysis [classical equations (4.32) and the equation of motion (1.80) with $p = 1$], the active phase mean-field phase corresponds to point B in Fig. 4.3. The system can be brought to extinction by tuning the control parameter $m = \sigma - \gamma$ to zero. The transition is represented by the phase portrait with the three zero-energy trajectories intersecting at the point $(1, 0)$. To focus on the vicinity of this point the shift of variable

$$p - 1 \to p \qquad (4.39)$$

Generalization of the Ginzburg-Landau potetntial Description to nonequilibrium (two fields)

PHYSICAL REVIEW E 74, 041101 (2006)

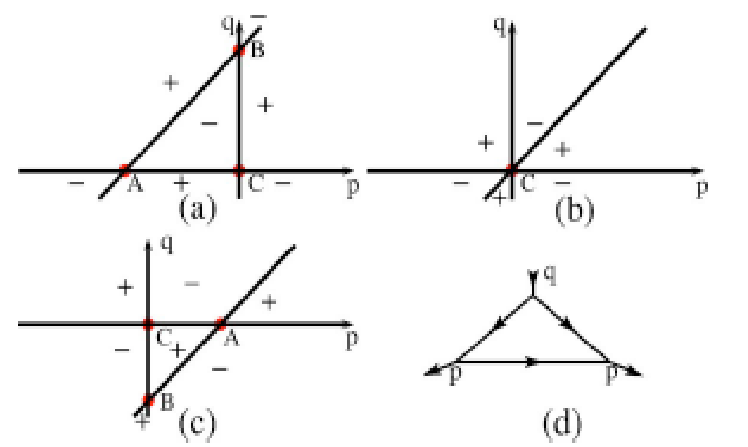
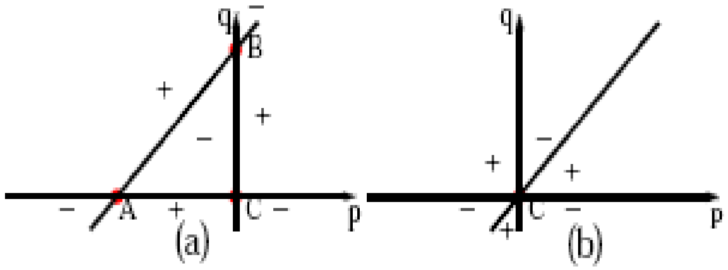


FIG. 5. (Color online) Generic phase portrait of DP models in the vicinity of the phase transition [after the shift, Eq. (30)]. (a) Active phase, $m>0$; (b) transition point, $m=0$; (c) extinction phase, $m<0$. The plus and minus signs show the sign of the Hamiltonian in each sector. (d) The one-loop diagram renormalizing u vertex (vertexes m and v are renormalized in a similar way).

Generalized, *n*-particle Contact Processes

- According to the phase-space description the triangular topology encodes DP behavior. For unary reaction models this is called the **DP hypothesis.**



- Tuning of simultaneous intersection of more than 3 lines → **multi-crtical points**
- In case of *n* particle reactions, when the lower levels are not generated by RG (fluctutions) the *q=0* line is „*n*" times degenerated → **different universality classes ?**
- Binary particle reaction models:

$$2A \xrightarrow{\lambda} \emptyset ; \quad 2A \xrightarrow{\sigma} 3A ; \quad 3A \xrightarrow{\mu} 2A , \tag{4.102}$$

where the last reaction is included (as usual in bosonic models) to prevent infinite proliferation of particles in the active phase. It is easy to construct the corresponding reaction Hamiltonian by the recipe given in Sect. 1.6.1.

$$\begin{aligned} H_R &= \frac{\lambda}{2}(1-p^2)\,q^2 + \frac{\sigma}{2}(p^3-p^2)\,q^2 + \frac{\mu}{6}(p^2-p^3)\,q^3 \\ &= \frac{1}{2}\left(\lambda\,(1+p) - \sigma\,p^2 + \frac{\mu}{3}\,p^2\,q\right)(1-p)\,q^2 . \end{aligned} \tag{4.103}$$

Binary particle reaction models (classes)

- Novel phase-space topology, novel class ? Long debates: PRG could not find corresponding stable fixed point (*Janssen et al*)
- The site restricted version does not require the *3A->2A* reaction, but an explicit diffusion of *A*-s, otherwise the *2A->3A, 2A->*0 (pair contact process) shows DP like behavior.
- The diffusive site restricted version the: PCPD shows diffusion dependent scaling behavior by numerical methods
- Some simulations claim that : PCPD ~ DP (since for D large there is a drift in the exponents)

The corresponding zero energy lines are shown on Fig. 4.13 with the $q =$

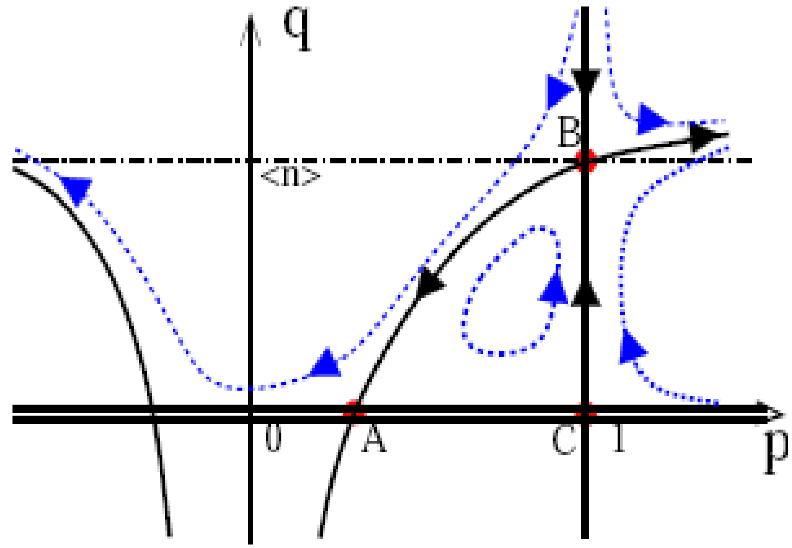
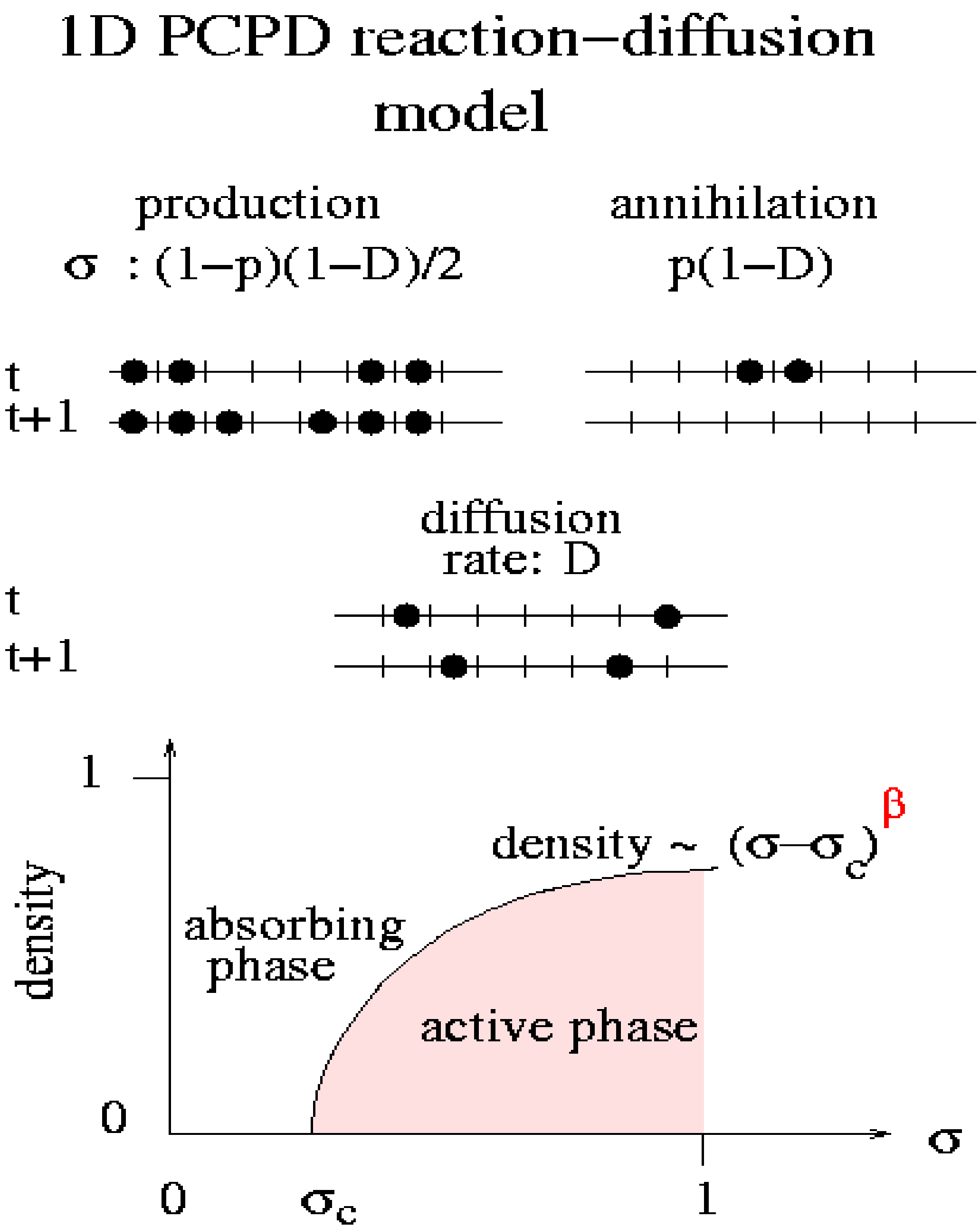


Fig. 4.13 Phase portrait of the annihilation-fission system in the active phase (cf. DP, Fig. 4.3). The zero-energy line $q = 0$ is doubly degenerate and is depicted by the double line. At the transition points A, B, and C coalesce. From [Elgart and Kamenev (2006)].

0 being double degenerated. The system is in the active phase for $\sigma > 2\lambda$ corresponding to point B (and particle density $\langle n \rangle$). By by tuning the control parameter $m = \sigma/2 - \lambda$ to zero the model can be driven to extinction. The $\sigma/2 = \lambda$ condition corresponds to a continuous phase transition. At the transition point the *four* zero-energy lines are intersecting in the point C.

Binary production (PCPD) model



- Two absorbing states **without symmetry,** one of them is diffusive. *(Carlon, Henkel, Schollwock (PRE 2001).*
- Bosonic field theory (*Howard&Tauber'97*) failed to describe critical behavior. In the bosonic model diverging active phase.
- Fermionic model shows different critical behavior but field theory is too hard. Numerical methods show new exponents.
- No extra symmetries or conservation laws of the action has been found to explain unexpceted critical behavior !

Mysteries of PCPD

- The upper critical dimension predicted:

  $d_c = 2(m-n-1)/(m-1) = \mathbf{2}$ $(d_c^{DP}=4)$
  confirmed by simulations (*Ódor et al 200*
- The mean-field exponents are different from those of DP : $\alpha = 1/n$
- For $d \geq 2$ **non-DP class,** why would it collapse at $d=1$ to DP ?
- Fermionic version and FT suggest that PCPD is better described by a multi-component model DP coupled to diffusing particles

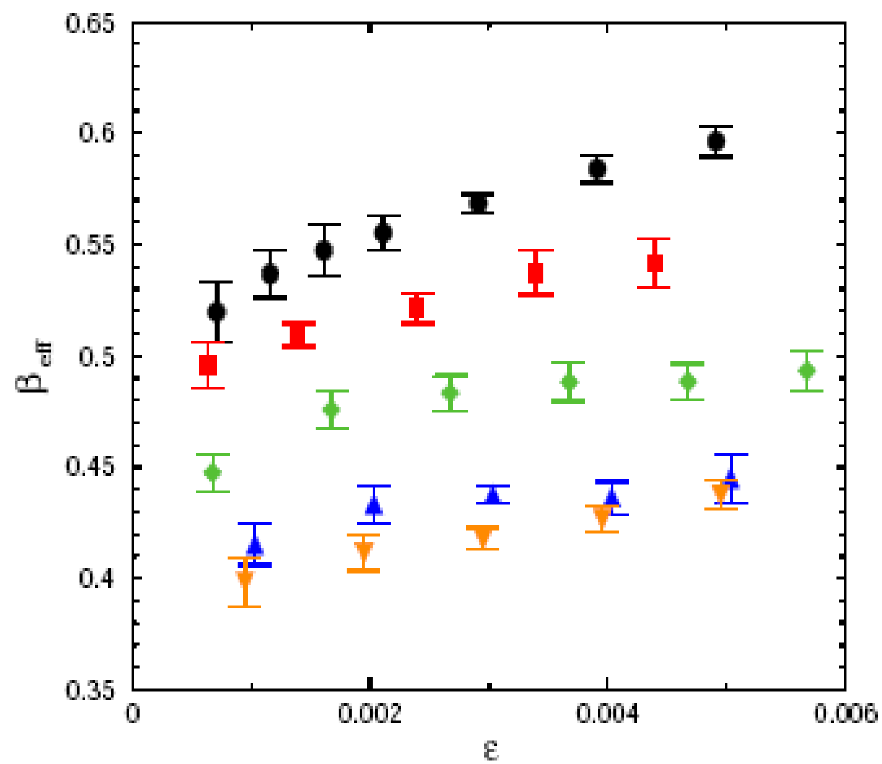
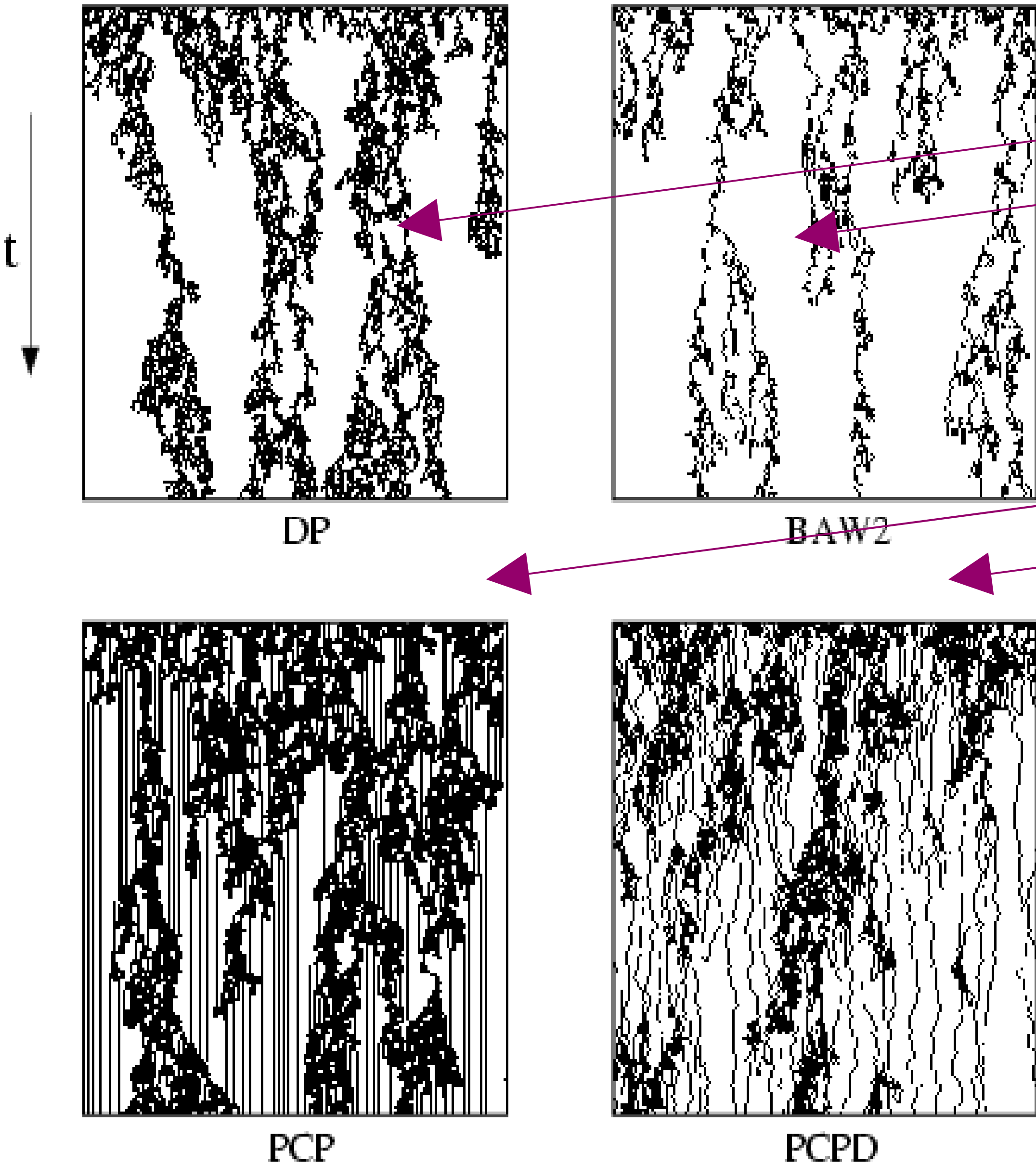


Fig. 6.5 Effective β exponents for different diffusion rates. The circles correspond to $D = 0.05$, the squares to $D = 0.1$ the diamonds to $D = 0.2$, the up-triangles to $D = 0.5$ and the down-triangles to $D = 0.7$. From [Ódor (2000)].

Space-time evolution of universal nonequilibrium spreading models with absorbing states in *1+1d*

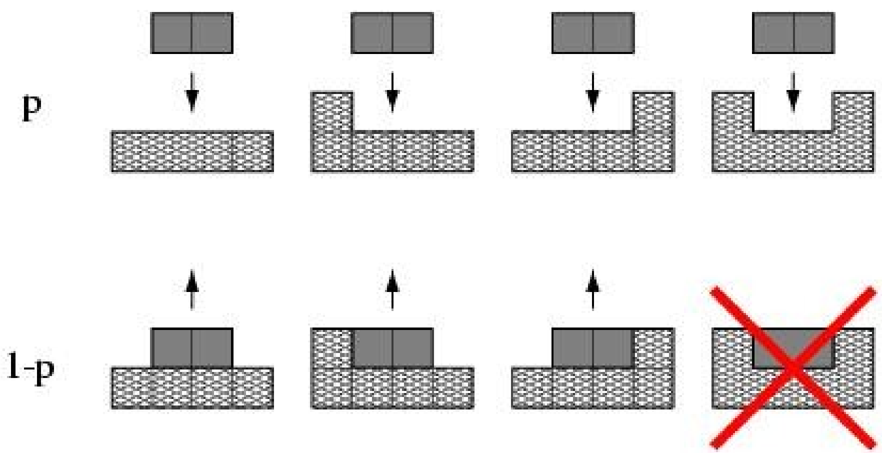


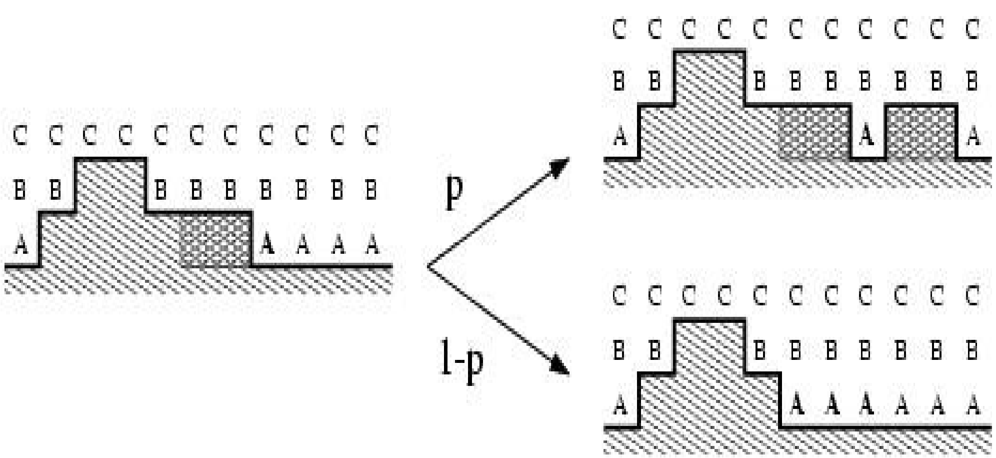
- **Unary** production **spreading without and with *parity conservation (PC)*:**
  $$A \rightarrow (m+1)A, \;\; 2A \rightarrow 0$$
- **Binary** production spreading coupled to slave modes without and with *diffusion*:

$$2A \rightarrow (m+2)A, \;\; 2A \rightarrow 0$$

Reactive and diffusive sectors, changing exponents by varying the diffusion rate.

Possible realization of PC by a surface growth model

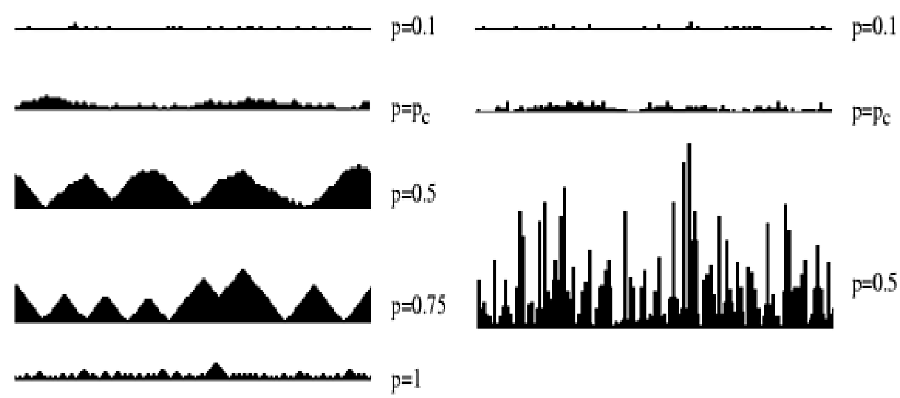




- Dimer growth model $\leftrightarrow$ Unidirectionally coupled, parity conserving RD systems:

$$A \rightarrow 3A \qquad B \rightarrow 3B \qquad C \rightarrow 3C \ldots$$
$$2A \rightarrow 0 \qquad 2B \rightarrow 0 \qquad 2C \rightarrow 0 \ldots$$
$$A \rightarrow A+B \qquad B \rightarrow B+C \qquad C \rightarrow C+D$$

- Growth transition $\leftrightarrow$ absorbing phase transition at each level with parity conserving class behavior

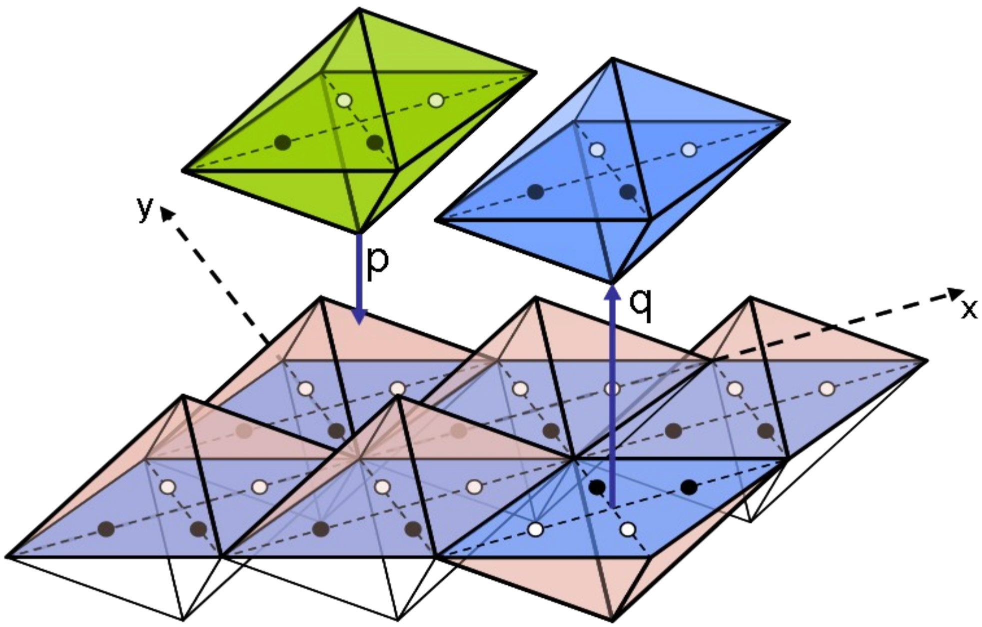


H. Hinrichsen, G. Ódor, PRL, PRE 1999.

Mapping to surfaces and interfaces

The morphology of a growing interface is usually characterized by its width

$$W(L,t) = \left[\frac{1}{L}\sum_i h_i^2(t) - \left(\frac{1}{L}\sum_i h_i(t)\right)^2\right]^{1/2}. \tag{7.2}$$



Mapping of two dimensional Kardar-Parisi-Zhang type surface growth onto driven lattice gas of dimers

G. Ódor et al PRE (2008)

- Technological point of view the control of their roughness is becoming critical for applications in fields such as microelectronics, image formation, surface coating or thin film growth
  (see: *T. S. Chow, Mesoscopic Physics of Complex Materials Texts in Contemporary Physics, Springer 2000)*

- Dynamics of a tumor growth ~ Molecular beam epitaxy class
  (B. Brutovsky, D. Horvath, V. Lisy, physics/0704.3138

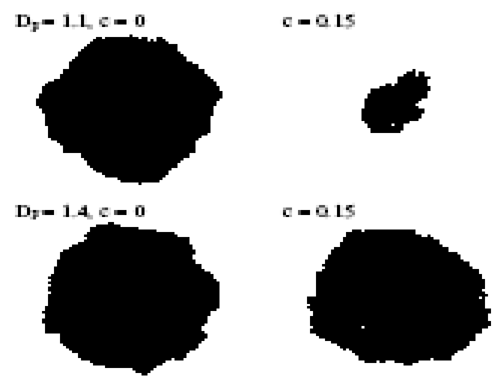


FIG. 6: Simulation of the impact of chemotherapy to the CA cluster after 200 time steps of the growth. The left column

Topological effects in low dimensions

- Hard-core exclusion among different species in 1d RD model:

  Pair annihilation : $AA \rightarrow 0$, $BB \rightarrow 0$

  Branching:

  $A \rightarrow BAB$ or $A \rightarrow ABB$

  different phase transitions
- This overwrites other symmetries and conservation laws
- Hypothesis:

  In one-dimensional, coupled branching and annihilating random walk systems of N-types of excluding particles at $\sigma_c = 0$ two universality classes exist, those of 2-BARW2s and 2-BARW2a models. *(Ódor PRE 2001)*
- Confirmed for binary models too..

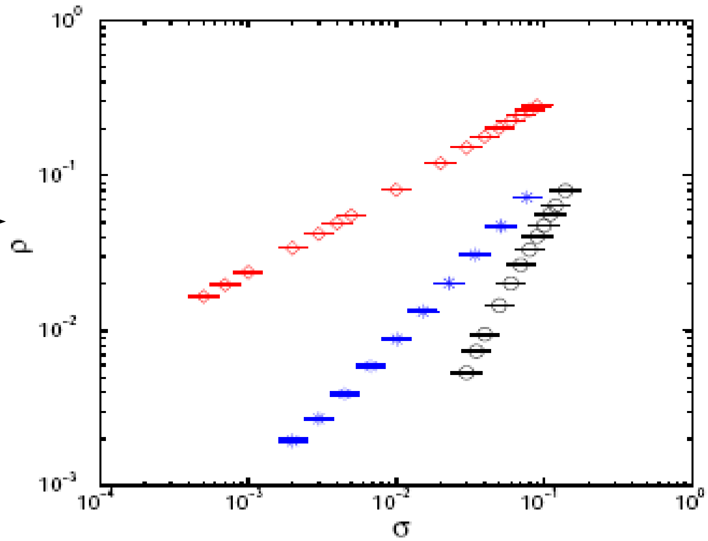


FIG. 10. Steady state density in the one-dimenional 2-BARW-2 model. Circles correspond to the asymmetric branching with $\beta = 2$, diamonds to the symmetric branching with $\beta = 1/2$ and stars to the model without exclusion ($\beta = 1$).

Critical universality classes summary

- Which factors determine the PT universality class of a model of short range interactions ?

  Besides the spatial dimensions, boundaries, inhomogenities:

  1) Mean-field classes of RD : $nA \rightarrow (n+k)A$, $mA \rightarrow (m\text{-}l)A$
  2) Symmetries, conservation laws like in equilibrium (BAW2 …)
  3) Initial conditions (temporal boundary condition)
  4) Topological effects in low dimensions (multi-comp systems...)
  5) Dynamically generated long range memory (coupled systems...)
  6) For competing dynamics diffusion can play a role

  See also: *G. Ódor, Rev. Mod. Phys. 76 (2004) 663.*

- Recent interests:

  The effects of
  long-range interactions,
  underlying networks