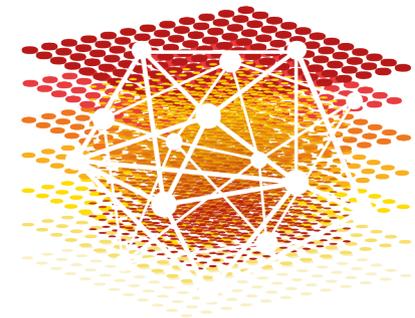


Heterogeneity and universality of power-grids

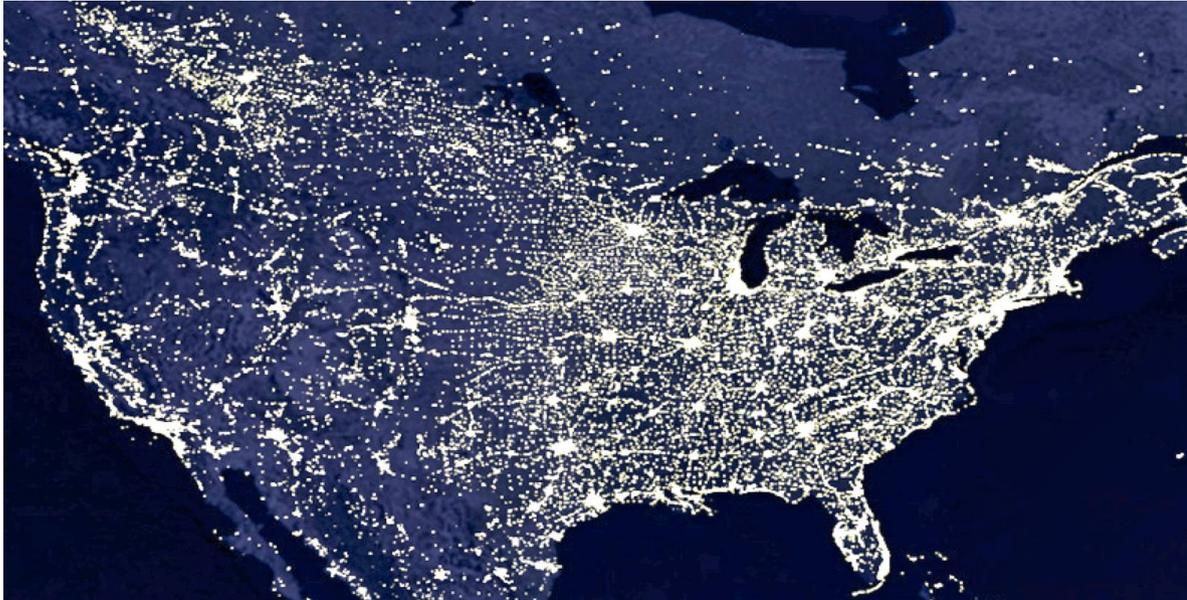


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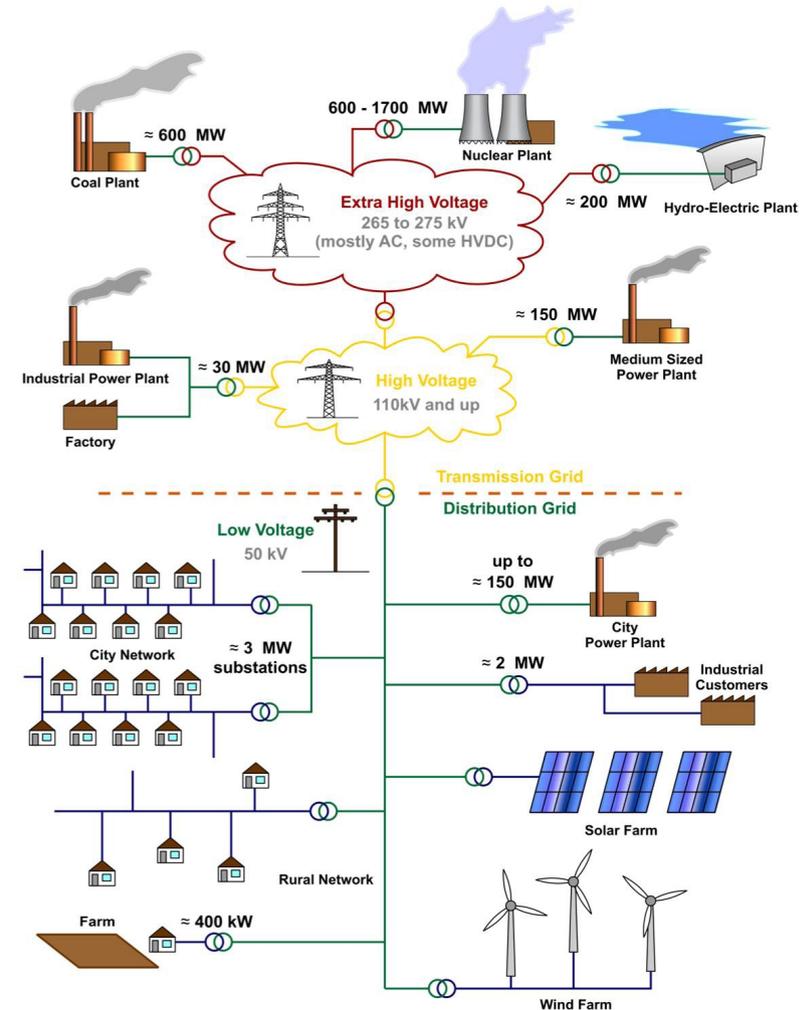
Géza Ódor, B. Hartmann, I. Papp, K. Benedek
Centre for Energy Research, Budapest



Electric Energy & Power Network



- Electric energy is critical for our technological civilization
- Purpose of electric power grid: generate/transmit/distribute
- Challenges: multiple scales, nonlinear, & complex system, growing number of renewables & deregulation



Largest man made machines
Synchronization for a whole
Continent is required

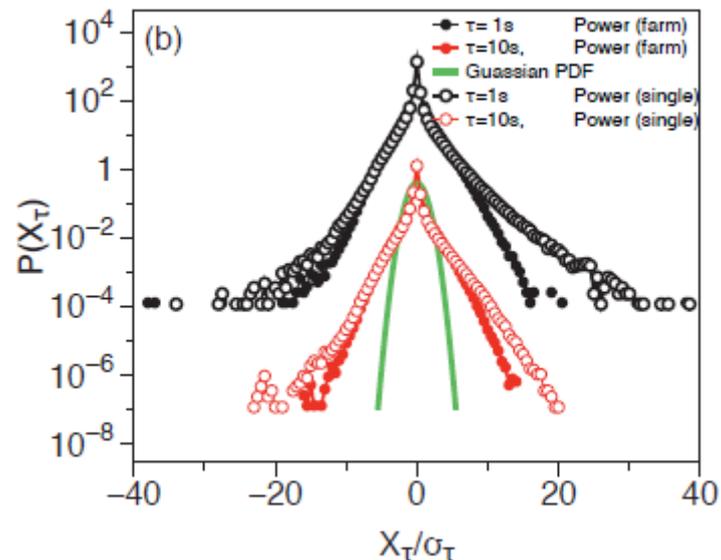
Some recent large-scale blackouts in the world and their consequences

No.	Country	Year	Load loss (GW)	Economic loss	People affected (*Million)	Duration (hours)	Reference
1	Iran	2003	~7	Not available	22	8	[10, 13]
2	USA, Canada	2003	61.8	\$ 6.4 billion	50	16-72 (USA), up to 192 (Canada)	[10-12]
3	Italy	2003	24	Over €120 million	~ 56	Up to ~18	[10, 12]
4	Russia	2005	~3.5	\$ 1-2 billion	4	~4	[46, 50]
5	Western Europe	2006	~14	Not available	15	~2	[12]
6	USA and Mexico	2011	4.3	Up to \$118 million	Over 5	~11	[50]
7	India	2012	~48	Not available	670	2-8	[13, 51]
8	Turkey	2015	32.2	Not available	70	More than 7	[13]

Renewable energy adds more instability due to the large fluctuations of the power sources

Anvari et al 2018

Complexity and Synergetics.



Scale-free blackout size distributions

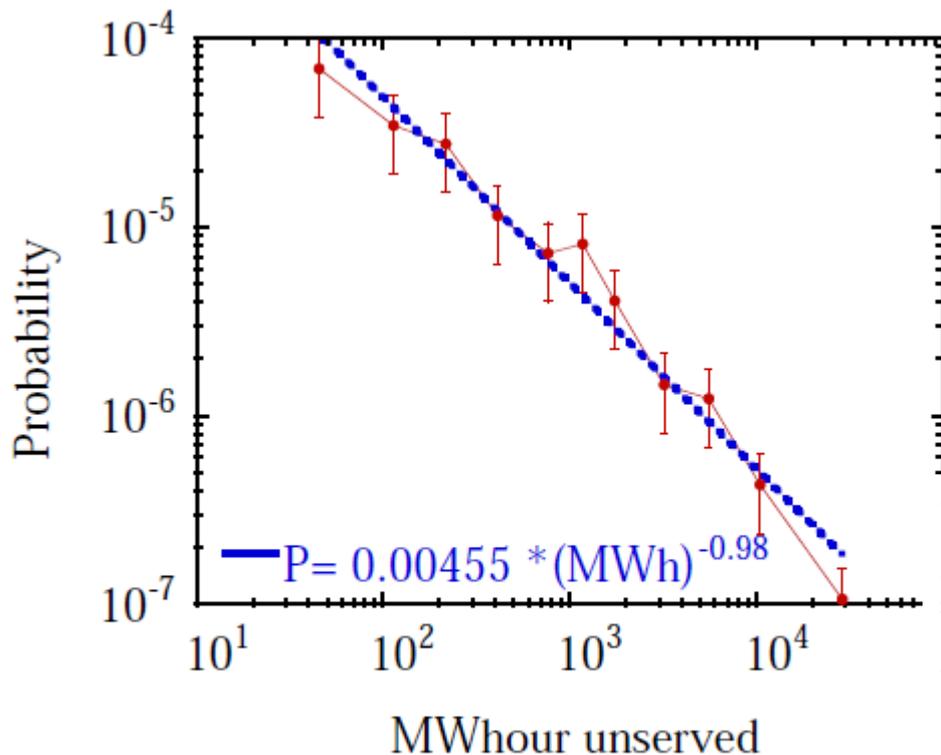


Figure 4. Probability distribution function of energy unserved for North American blackouts 1993-1998.

Self-Organized Criticality (SOC)

describes this:

Competition of supply and demand

Universality (in grid characteristics) ?

B. Carreras et al, Proceedings of Hawaii International Conference on System Sciences, Jan. 4-7, 2000, Maui, Hawaii. 2000 IEEE

Extreme events occur more frequently than by Gaussian model prediction

TABLE I. Observed and simulated power law exponents in the noncumulative pdf of blackout size. The power law exponent is often calculated by subtracting one from an estimate of the slope of a log-log plot of a complementary cumulative probability distribution.

Source	Exponent	Quantity
North America data (Ref. 6)	-1.3 to -2.0	Various
North America data (Refs. 19 and 20)	-2.0	Power
Sweden data (Ref. 21)	-1.6	Energy
Norway data (Ref. 22)	-1.7	Power
New Zealand data (Ref. 23)	-1.6	Energy
China data (Ref. 24)	-1.8	Energy
OPA model on tree-like 382-node (Ref. 8)	-1.9	Power
Hidden failure model on WSCC 179-node (Ref. 9)	-1.6	Power
Manchester model on 1000-node (Ref. 10)	-1.5	Energy
CASCADE model (Ref. 11)	-1.4	No. of failures
Branching process model (Ref. 12)	-1.5	No. of failures

Outage durations with PL tails

Universality ?

Electrical outages \neq Blackout cascades, still they show PL duration tails:

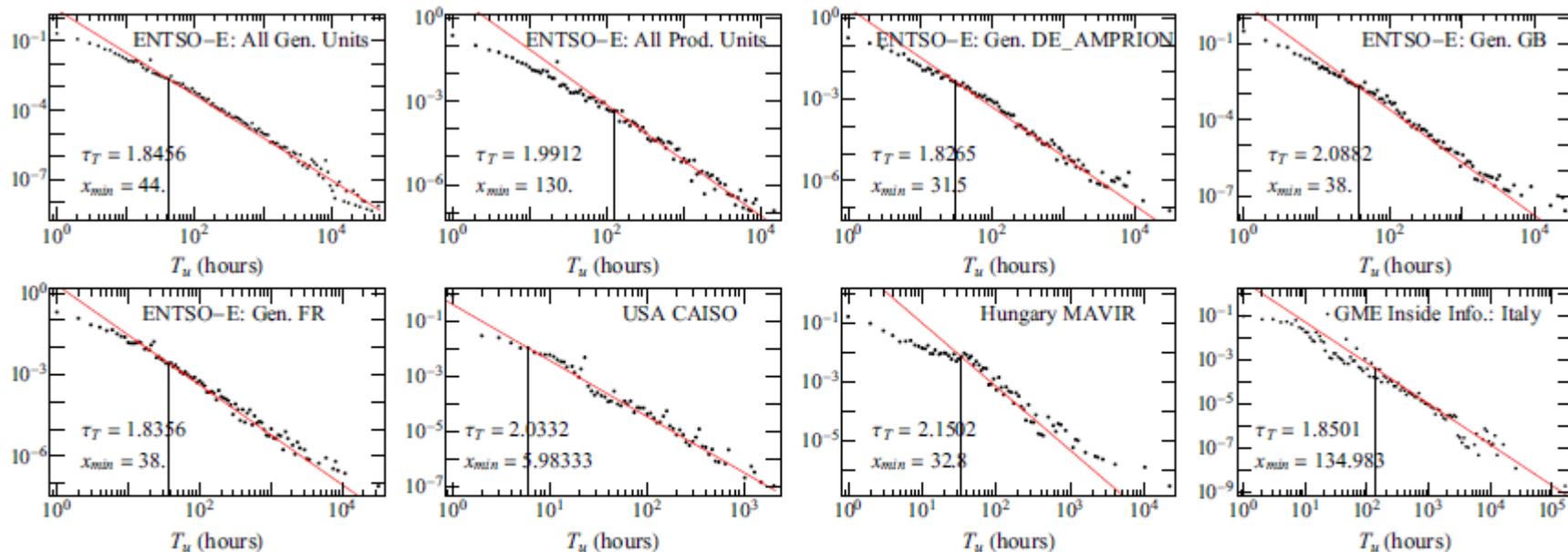


FIG. 2. Probability distributions (black dots) of generation outages measured in terms of the unavailable duration. For the ENTSO-E data, we show the generation outage data for the control areas “DE_AMPRION”, “GB”, and “FR”, as well as the generation and production outage data from all control areas. The fitted power laws and their corresponding x_{min} values are marked by solid red lines and vertical black lines, respectively.

Following power-spectral analysis we proposed SOC and HOT models to understand
See our recent paper : [PRX Energy 2 \(2023\).033007](#)

The EU 2016 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data

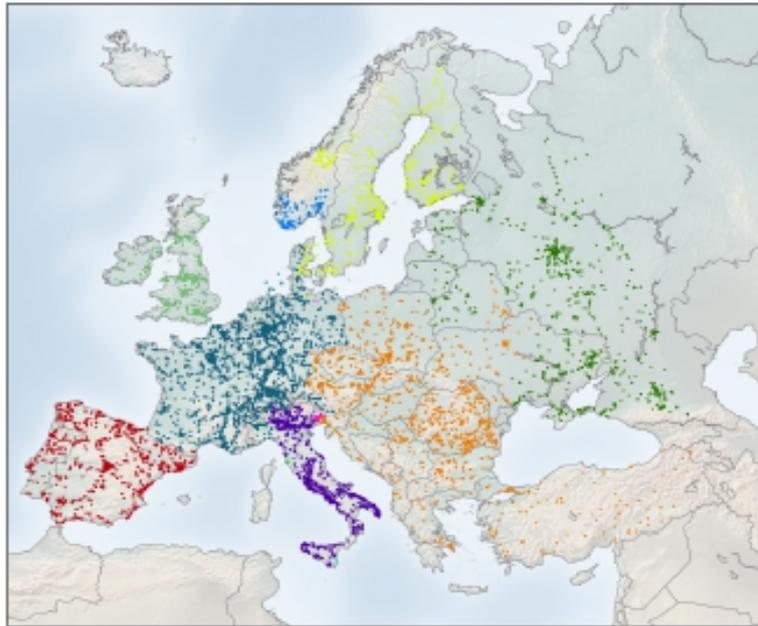
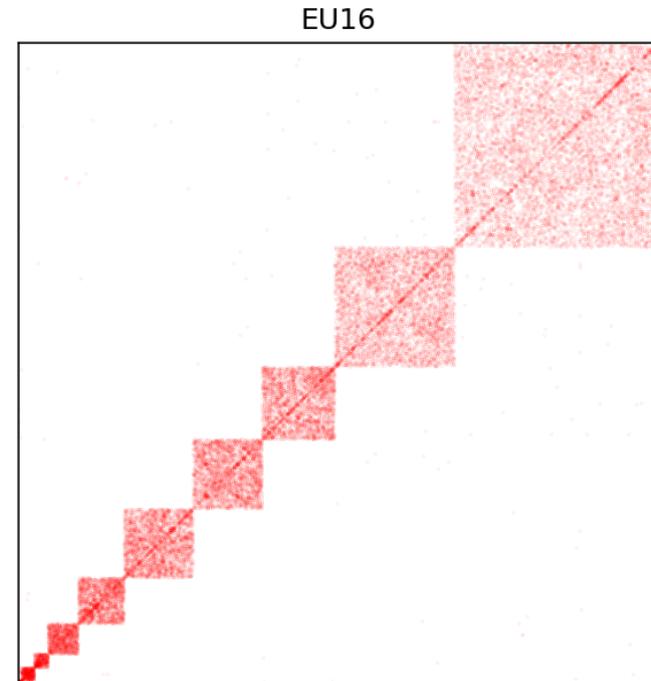


FIG. 9. All nodes of the European power-grid 2016 data separated into 12 communities, taking into account admittance using a giant component of 13 478 nodes connected by 18 395 links, maintaining the modularity score close to the maximum $Q \approx 0.795$.



Adjacency Matrix

Graph dimension: $\langle N_r \rangle \sim r^d$,

Modular HV network, with graph dimension $d = 2.6(1)$

The EU 2022 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data

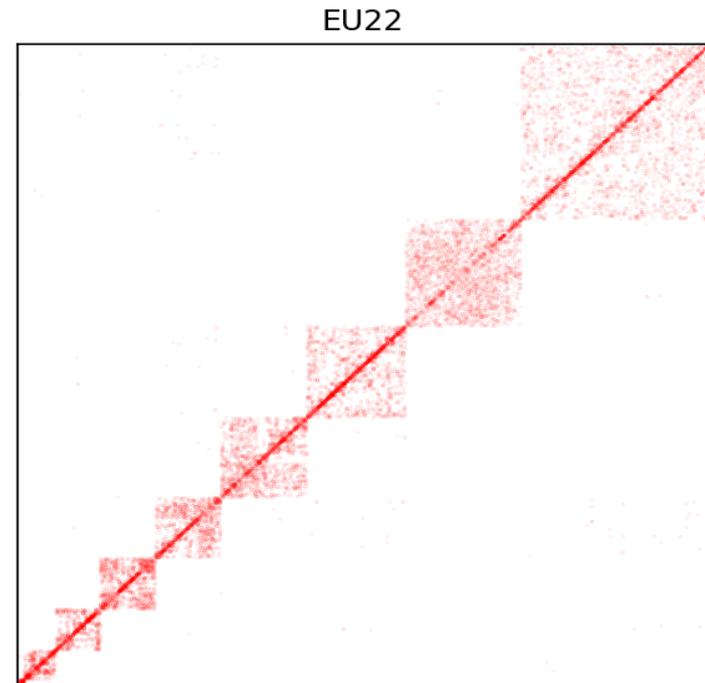
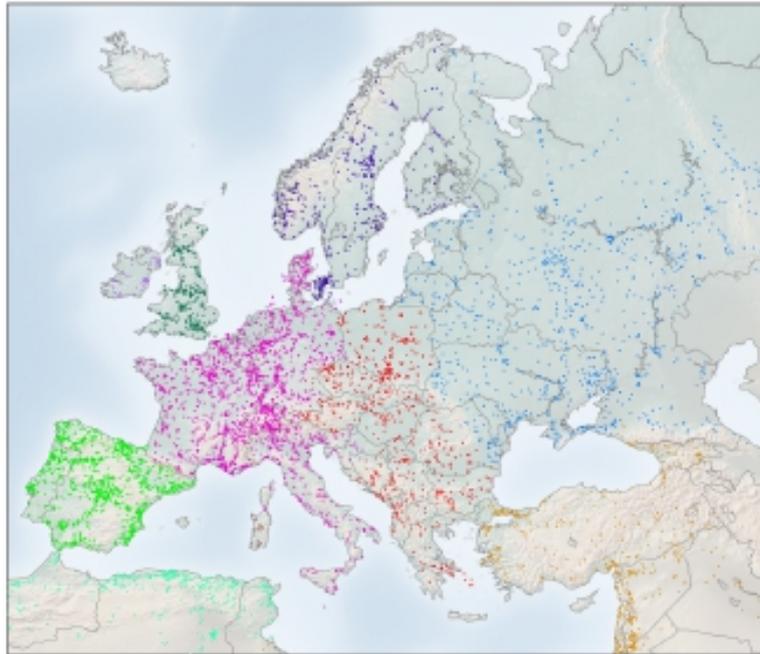


FIG. 11. All nodes of the European power-grid 2022 data giant component, separated into 10 communities, taking into account the admittances and 7411 nodes connected by 10 912 edges without smaller voltage level edges, maintaining the modularity score $Q \approx 0.854$.

Adjacency Matrix

Modular HV network, with graph dimension: $d = 1.8(2)$

The US 2016 HV network

SciGRID project based on ENTSO-E & OpenStreetMap data

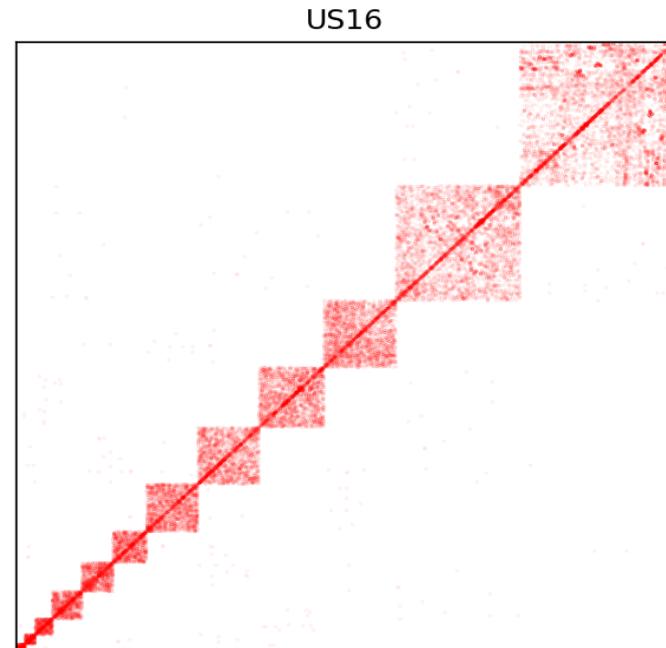


FIG. 12. All nodes of the USA power-grid 2016 data grid component, separated into 12 communities, taking into account the admittances and 14 990 nodes connected by 20 880 edges, maintaining the modularity score $Q \approx 0.859$ with resolution $\Gamma = 1 \times 10^{-4}$.

Adjacency Matrix

Modular HV network, with with graph dimension $d = 2.4(1)$

Summary of community results

Community	Size (EU22)	$\langle k \rangle$ (EU22)	Size (EU16)	$\langle k \rangle$ (EU16)	Size (US16)	$\langle k \rangle$ (US16)
1	924	2.72	4285	2.83	3511	2.79
2	479	2.70	2526	2.66	2829	2.98
3	2016	2.84	1527	2.67	1640	2.72
4	698	3.06	1461	2.72	1484	2.69
5	595	2.94	1455	2.69	1396	2.93
6	1059	2.66	966	2.77	1165	2.58
7	1237	2.68	638	2.57	768	2.97
8	16	2.81	289	2.06	710	2.57
9	332	2.18	277	2.99	673	2.70
10	55	2.74	26	3.07	390	2.84
11	-	-	22	3.31	230	2.43
12	-	-	6	2.66	194	2.69

TABLE I. Community sizes and average degrees for different data-sets, for the resolution $\Gamma = 10^{-4}$. We refer to sizes here as number of nodes in the respecting community. These structures correspond to the maps plotted on Figs.9, 11, 12.

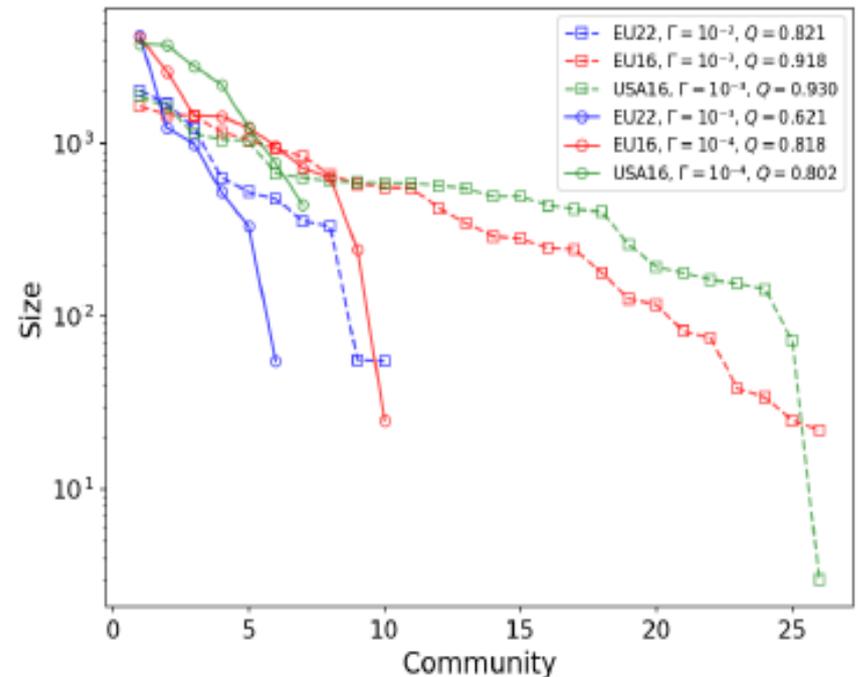


FIG. 8. Community size distributions at different Γ resolution parameters for different networks shown in the legend.

Louvain algorithm used with resolution
Parameter Γ

$$Q = \frac{1}{N \langle k \rangle} \sum_{ij} \left(A_{ij} - \Gamma \frac{k_i k_j}{N \langle k \rangle} \right) \delta(g_i, g_j),$$

Similar size distributions

Summary of network invariants

Graph: $G = (V, E)$ N nodes, E edges

Average degree $\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i$

Cumulative degree distribution $P(k_i > K) = C \cdot e^{-\frac{k_i}{\gamma_c}}$

Shortest path-length $L = \frac{1}{N(N-1)} \sum_{j \neq i} d(i, j)$, $L_r = \frac{\ln N - 0,5772}{\ln \langle k \rangle} + 1/2$

Clustering coefficient $C = \frac{1}{N} \sum_i \frac{2n_i}{k_i} (k_i - 1)$

Small world coefficient $\sigma = \frac{C/C_r}{L/L_r}$ Modularity $Q = \frac{1}{N \langle k \rangle} \sum_{ij} \left(A_{ij} - \Gamma \frac{k_i k_j}{N \langle k \rangle} \right) \delta(g_i, g_j)$.

Network	E	N	$\langle k \rangle$	γ_c	Q	Community #	L	L_r	C	C_r	σ
EU16	18393	13478	2,729	1,504	0,924	28	49,50	9,396	0,099	0,000203	92,702
EU22	10298	7411	2,779	1,640	0,849	12	46,83	8,653	0,098	0,000375	48,420
USA16	20880	14990	2,786	1,548	0,927	22	47,50	9,321	0,102	0,000186	107,785

Similar invariants, small world networks $\leftrightarrow d < 3$

Load and generator power distributions

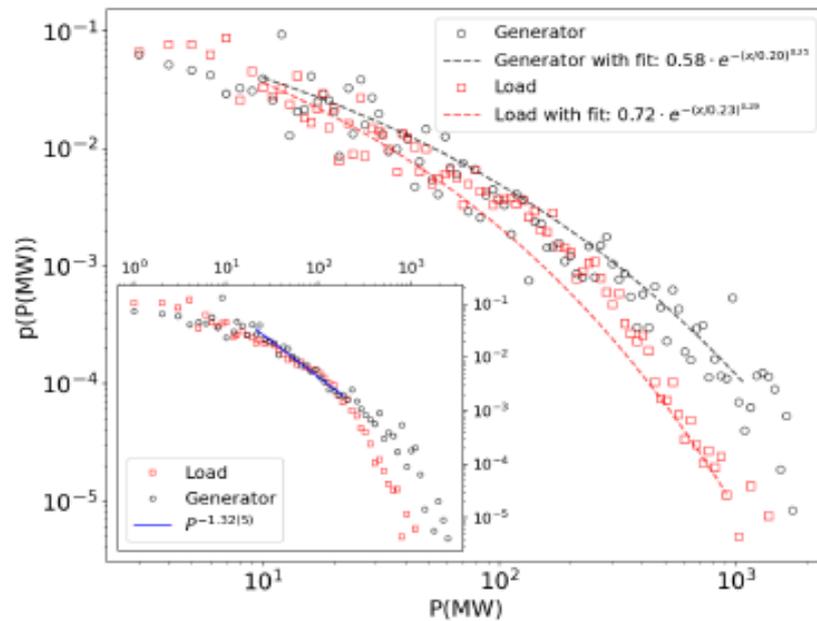


FIG. 5. Distribution of nodal generations and loads of the ENTSO-E 2016 database. Power-law fits were applied to the [20...300] MW range in the inset figure. The exponents of the fits are: $y = 1.16(5)$ both for generation and load curves, respectively. The load data shows an earlier size cutoff, which is an important characteristic of traditional power systems, where energy is produced in a centralized manner by large power plants to increase efficiency, and energy is consumed in a distributed manner. The main figure shows the same data, with stretched exponential fits, according to Eq. (7) in the range [10...1000] MW

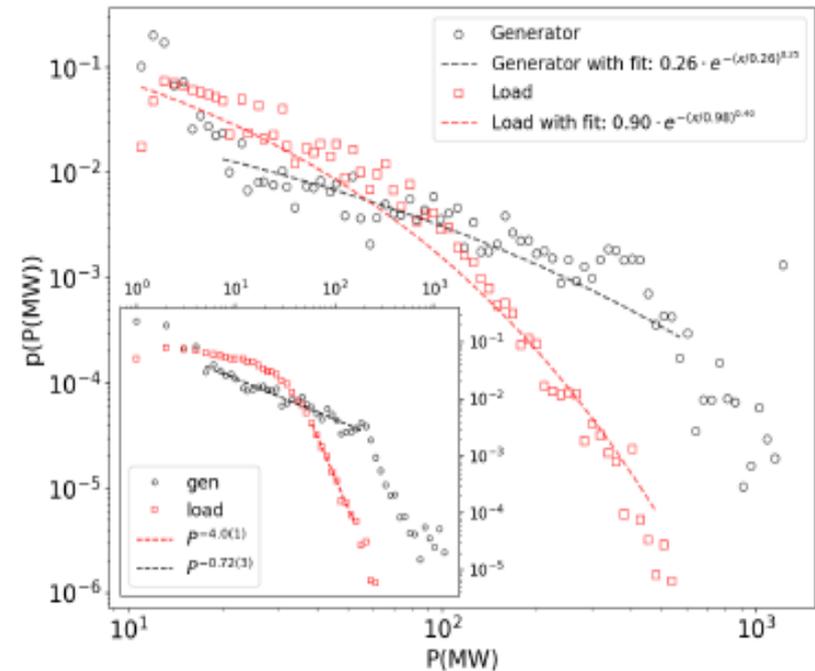


FIG. 6. Distribution of nodal generations and loads of the 2021 US [69] database. Inset: different power-law fits were applied to the [5...200] MW for generators and [50...200] MW for loads. The load data shows an earlier size cutoff as for the European case. The main figure shows the same data with stretched exponential fit according to Eq. (7) in the range [20...500] MW.

$$p(k) \propto \exp\left(-\left(k/B\right)^\beta\right) \quad \beta \sim 0.25 \text{ Universal?}$$

Admittances (interaction weights), calculated from cable lengths and specific resistances

$$R_{ij} = \left(\frac{U_c}{U_{ij}}\right)^2 \cdot L_{ij} \cdot R_{c_k} \quad P_{ij} = P_{c_k}$$

$$X_{ij} = \left(\frac{U_c}{U_{ij}}\right)^2 \cdot L_{ij} \cdot X_{c_k} \quad W_{ij} = \frac{P_{ij}}{X_{ij}} / \left\langle \frac{P}{X} \right\rangle$$

TABLE II. Characteristic values of relevant physical quantities in the modeled European power grids.

Voltage [kV]	R_c [Ω /km]	X_c [Ω /km]	C_c [nF/km]	P_c [MW]
120	0.0293	0.1964	9.4	170
220	0.0293	0.2085	9.0	360
380/400	0.0286	0.3384	10.8	1300

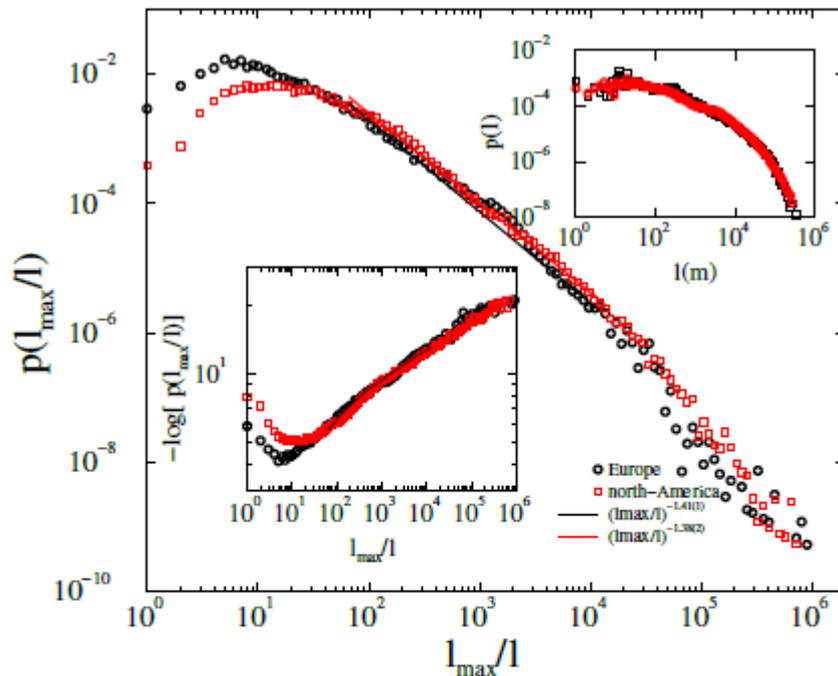
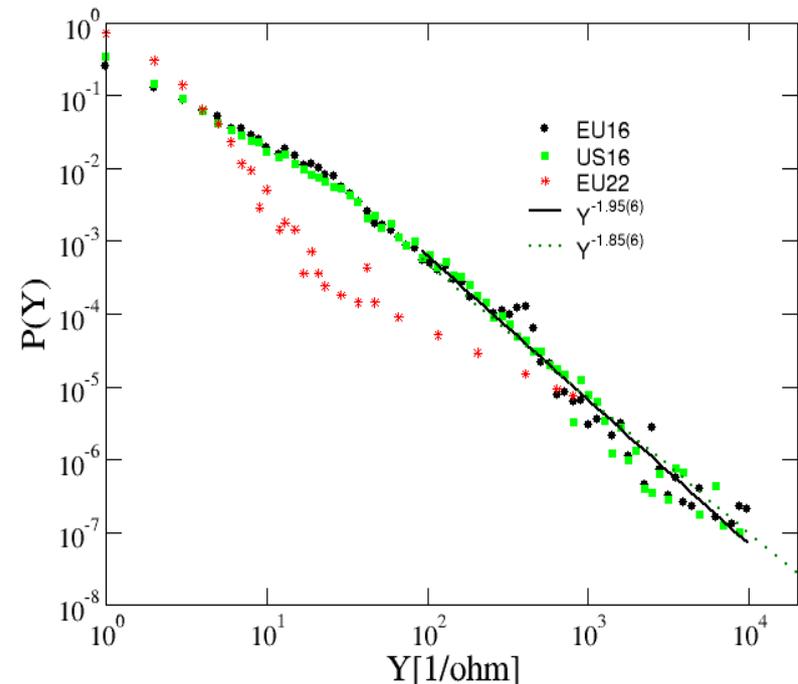


FIG. 14. Probability distributions of the inverse of cable lengths of the European and North-American SciGRID networks. Left inset: the same data plotted on the $-\ln(p)$ scale to compare with the stretched exponential assumption, that would correspond to a straight line tail, Right inset: probability distributions of the line lengths in meters.



PL exponents ~ 2 , **Universal ?**

Conclusions

Empirical power-grid networks are heterogeneous, still for $N \rightarrow \infty$,
i.e. on continent level, show \sim universal graph, electric measures

Short ($\sim 1m$) HV cable lengths \rightarrow fat tailed, power-law admittances
as graph edge weights

Self Organized Criticality ?

Cascade failures exhibit

Universal PL duration statistics

Thank you for the attention!

Recent, related publications:

Géza Ódor, Shengfeng Deng, Synchronization Transition of the Second-Order Kuramoto Model on Lattices

Entropy 25 (2023), 164

Géza Ódor, Shengfeng Deng, Balint Hartmann and Jeffrey Kelling
Synchronization dynamics on power grids in Europe and the United States,
Physical Review E 106 (2022) 034311.

Géza Ódor and Bálint Hartmann
Power-Law Distributions of Dynamic Cascade Failures in Power-Grid Models, *Entropy* 22 (2020) 666

Géza Ódor and Bálint Hartmann,
Heterogeneity effects in power grid network models
Phys. Rev. E 98 (2018) 022305

arXiv:2308.15326, arXiv:2310.09042