Phase transitions in binary, triplet and quadruplet reaction-diffusion systems

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1. Mean-field classes of reaction-diffusion systems
2. Binary production : 2A -> 3A, 2A -> 0 (PCPD)
3. Binary production: 2A -> 4A, 4A -> 2A
4. Binary production: 2A -> 3A, 4A -> 0
5. Triplet production: 3A -> 4A, 3A -> 2A
6. Triplet production: 3A -> 6A, 3A -> 0
7. Triplet production: 3A -> 5A, 2A -> 0
8. Quadruplet production: 4A -> 5A, 4A -> 0

9/17/03

Introduction

- Novel critical behavior has been found in case of non-unary production systems with diffusive particles (Mendes, Grassberger, Howard, Tauber, Carlon, Henkel, Schollwöck, Hinrichsen, Chaté, Noh, Park, Ódor ...)
- Common feature: No extra symmetry, no direct channel to absorbing state (A -/-> 0) like in case of DP. (Coupled syst.)
- Phenomenological Langevin by Hinrichsen predicts:

  $$d_c = 2 + (4 - 2\mu)/n \, , 1 \le \mu \le n$$

  but the validity of such formalism is debated for these models.
- Kockelkoren and Chaté proposed a table of universality classes in 1d, based on simulation of a supressed bosonic CA

Mean-field classes of general, one-component reaction-diffusion models

- Lattice models (site restricted) of the form:

$$\rho(t) \propto t^{-\alpha} \qquad \rho(\infty) \propto \epsilon^{\beta}$$

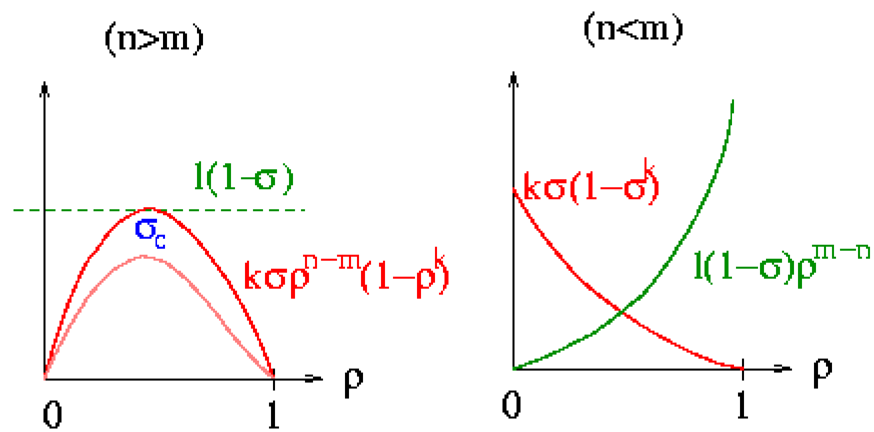
$$n A \xrightarrow{\sigma} (n+k) A \qquad m A \xrightarrow{\lambda} (m-l) A$$

(n>1, m>1, k>0, l >0, m -l ≥0) $\partial \rho / \partial t = a k \sigma \rho^{n} (1-\rho)^{k} - a l (1-\sigma) \rho^{m}$

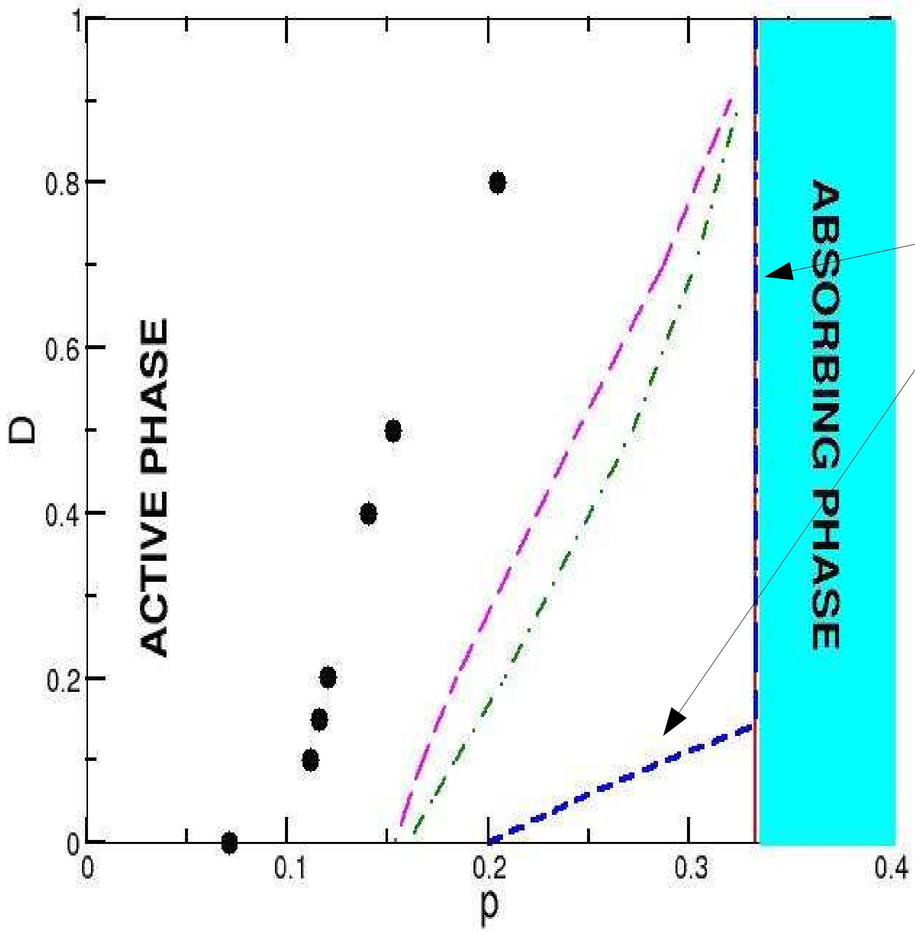
- The n = m case : $\beta = 1, \alpha = 1/n$ at: $\sigma_c = l/(k+l)$!!
- The n < m case : $\beta = 1/(m-n), \alpha = 1/(m-1)$ at: $\sigma_c = 0$
- The n > m case : First order transition

n and m determine the classes !

G. Ódor: PRE 67, 056114 (2003)
G. Ódor, Rev. Mod. Phys. (2004)



Cluster mean-field results for the 2A -> 3A, 2A -> 0 (PCPD) model



- Pair mean-field gives two different MF regions with:

  $\alpha = 1\,, \beta_2 = 1\,, \alpha = 1/2\,, \beta_2 = 2$

  (Carlon et al. 2000)

- 3 , 4 cluster approximations show single class.

  G. Ódor, PRE 67,016111 (2003)

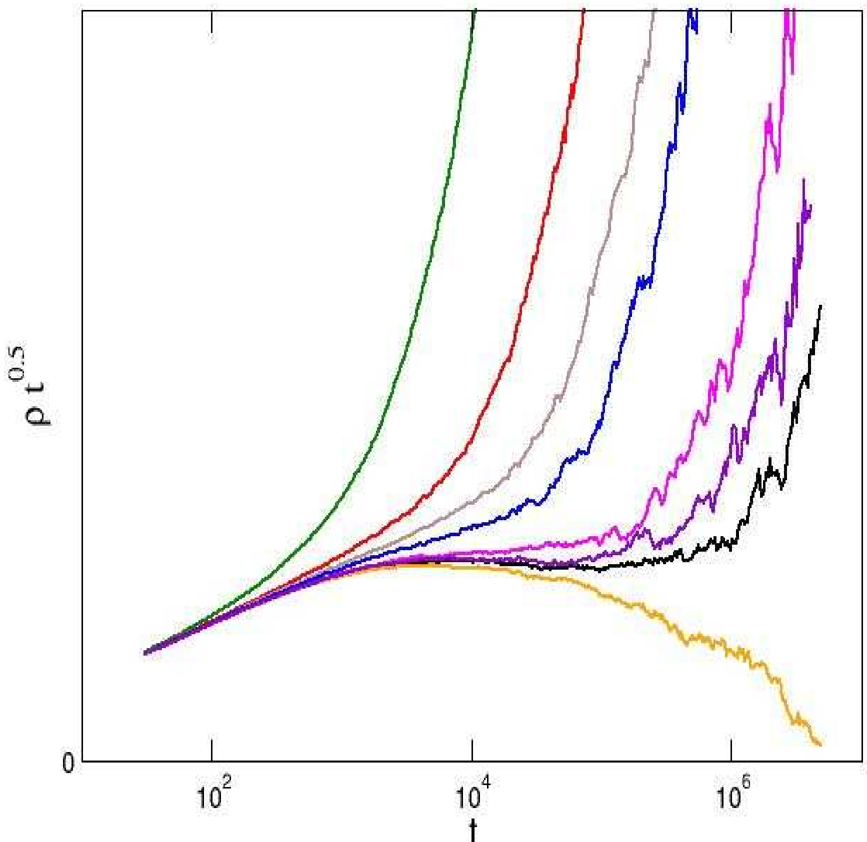
- MC and DMRG finds matching phase transition lines, but DMRG predicted:

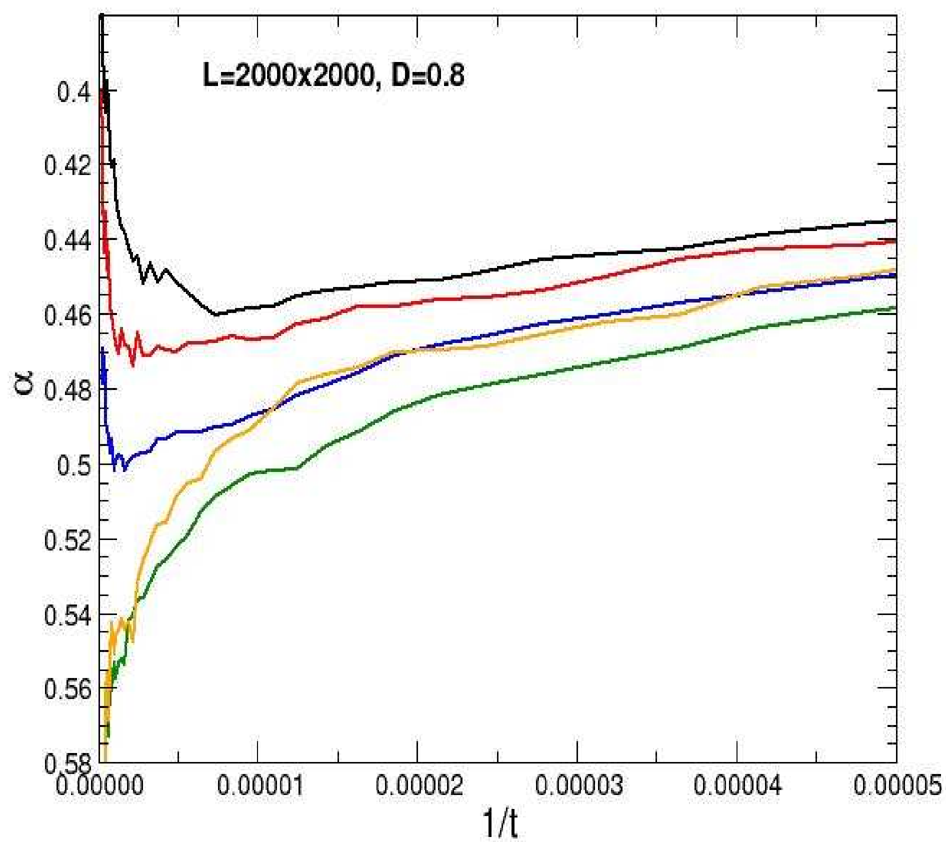
  PC class (Carlon et al. 2000),

  DP class (Barkema et al. 2003)

Simulation results for the 2A -> 3A, 2A -> A (site restricted) model in 2 dimensions

Mean-field density decay from homogenous state with : α=0.5
Logarithmic corrections, upper critical dimension = 2

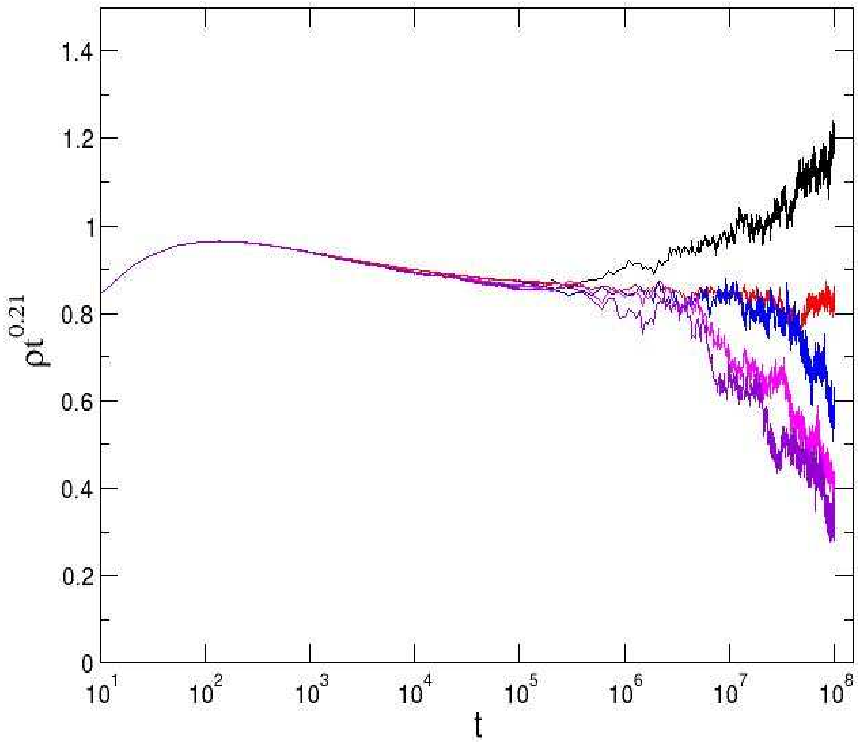


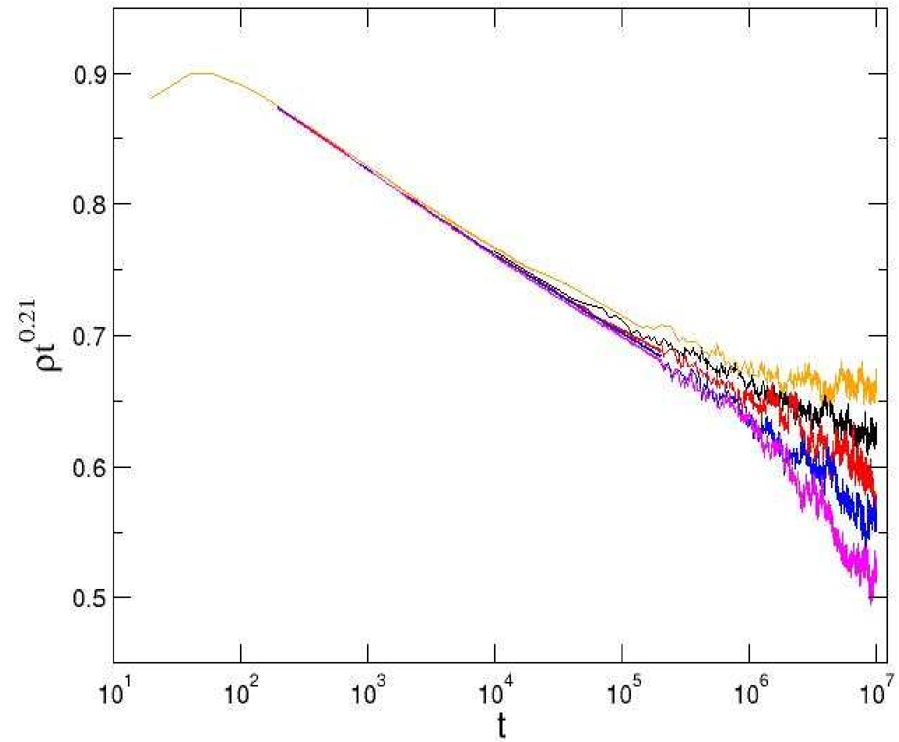


See also: *G. Ódor et al. PRE 65. 056113 (2002)*

Simulation results for the 2A -> 3A, 2A -> 0 (site restricted) model in 1 dimension

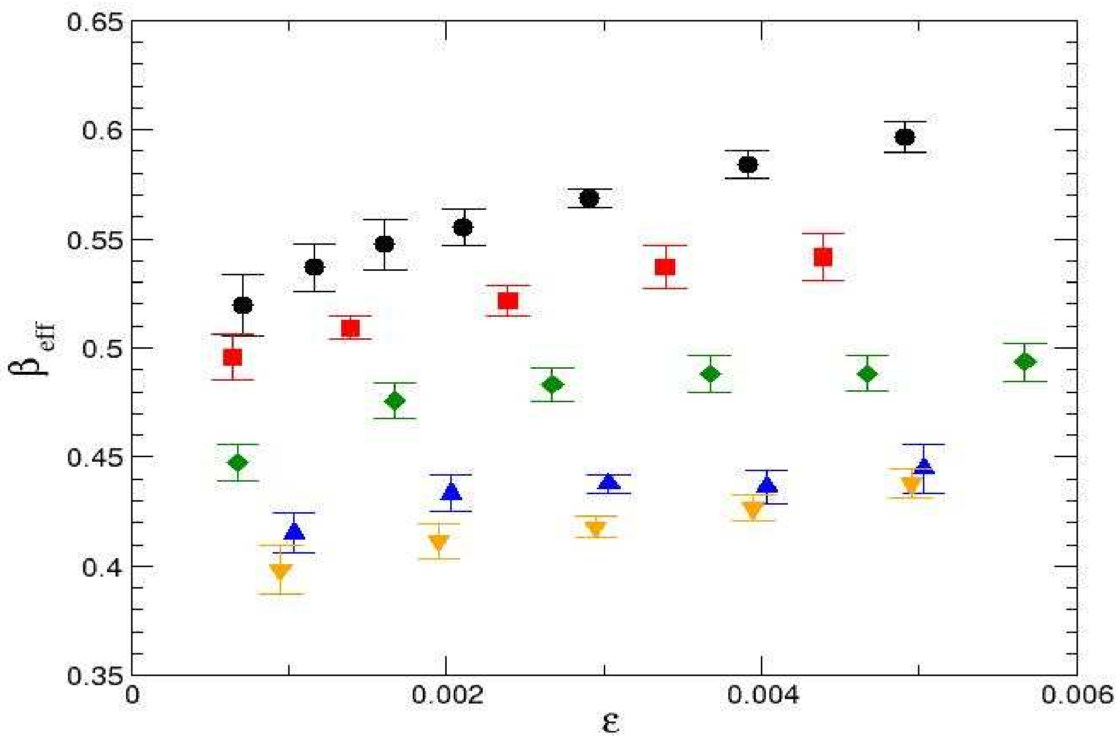
Density decay from homogenous state for: D > 0.2 with α=0.21,
and for D < 0.05 with α=0.21? Logarithmic corrections ?
G. Ódor, PRE 67, 016111 (2003). Agreement with Chaté et al.....





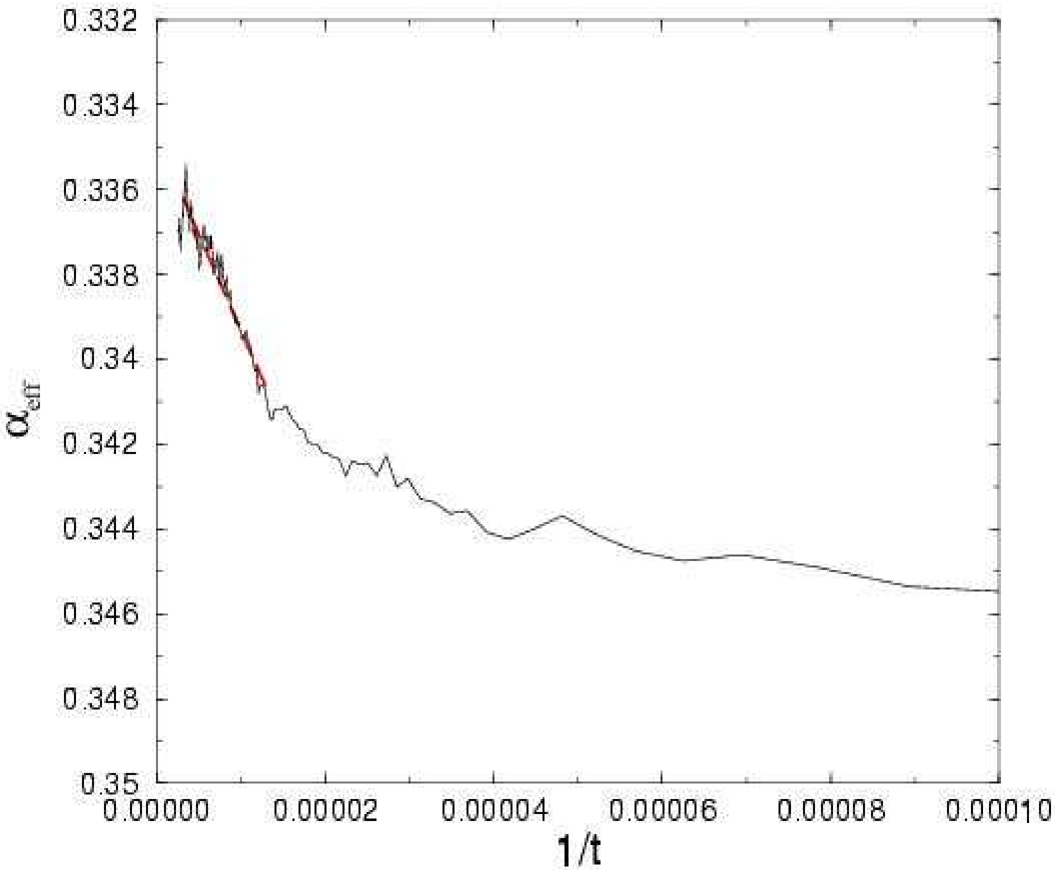
Simulation results for the 2A -> 3A, 2A -> 0 (site restricted) model in 1 dimension

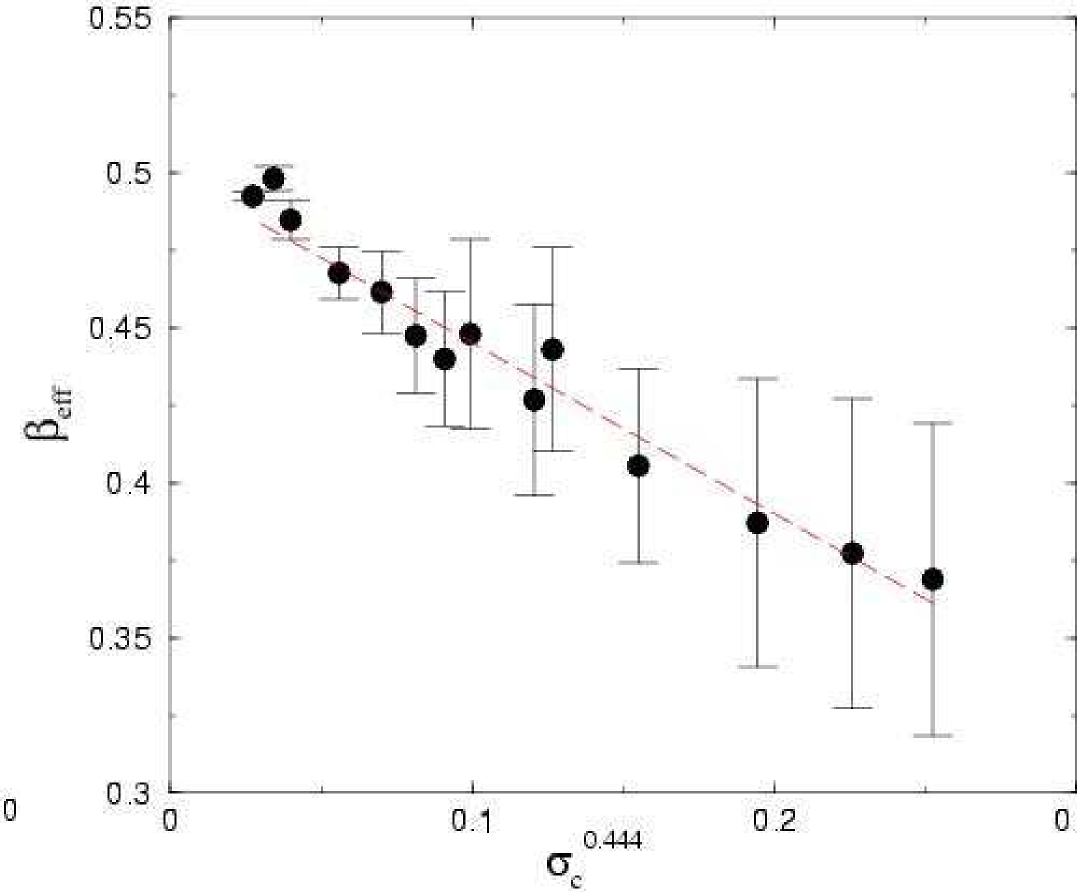
Steady state density for: D = 0.05, 0.1, 0.2, 0.5, 0.7. $\rho \propto \epsilon^{\beta}$
Logarithmic corrections may axplain the differences ... β = 0.40(2)
G. Ódor, PRE 67, 016111 (2003). Agreement with Chaté et al ...



Simulation results for the 2A -> 4A, 4A -> 2A (site restricted) model in 2 dimensions

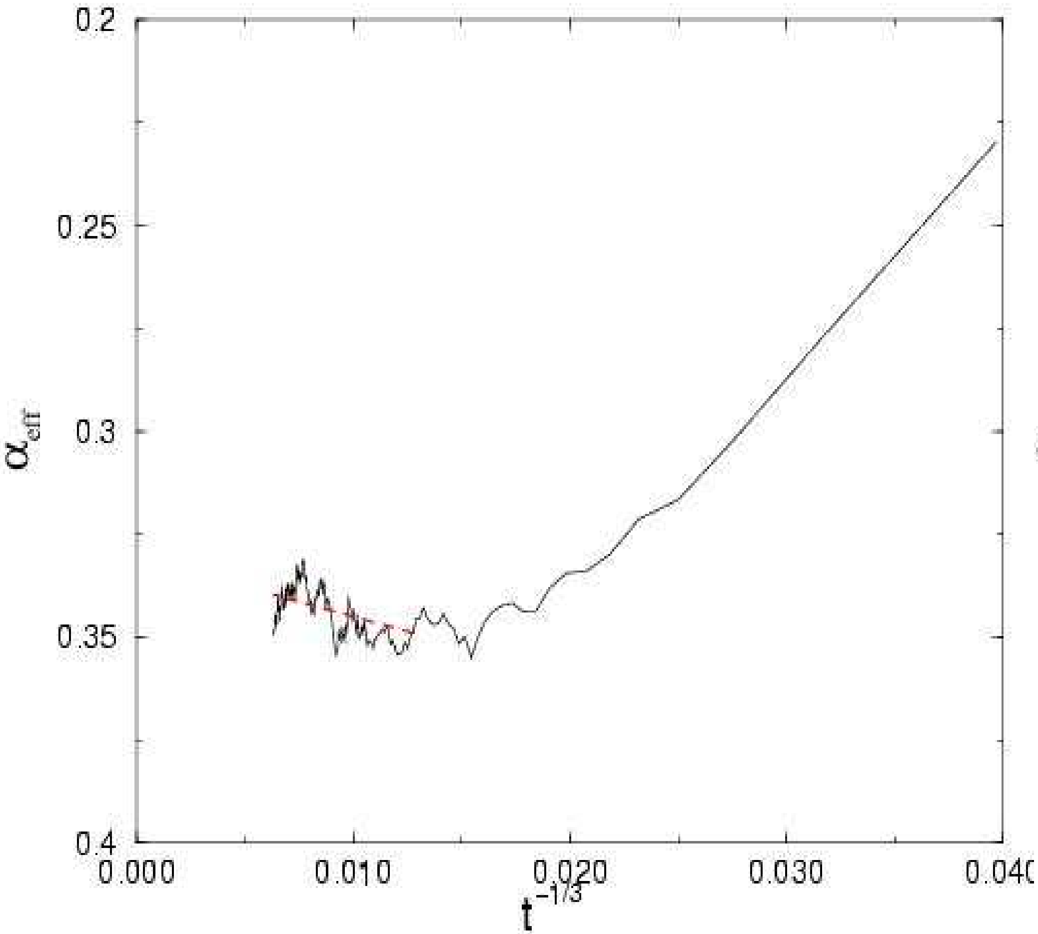
Mean-field type density decay and steady state behavior at D = 0.5 with α=1/3 and β=1/2 at zero branching rate. *Braz. J. of Phys. 33, 431 (2003)*

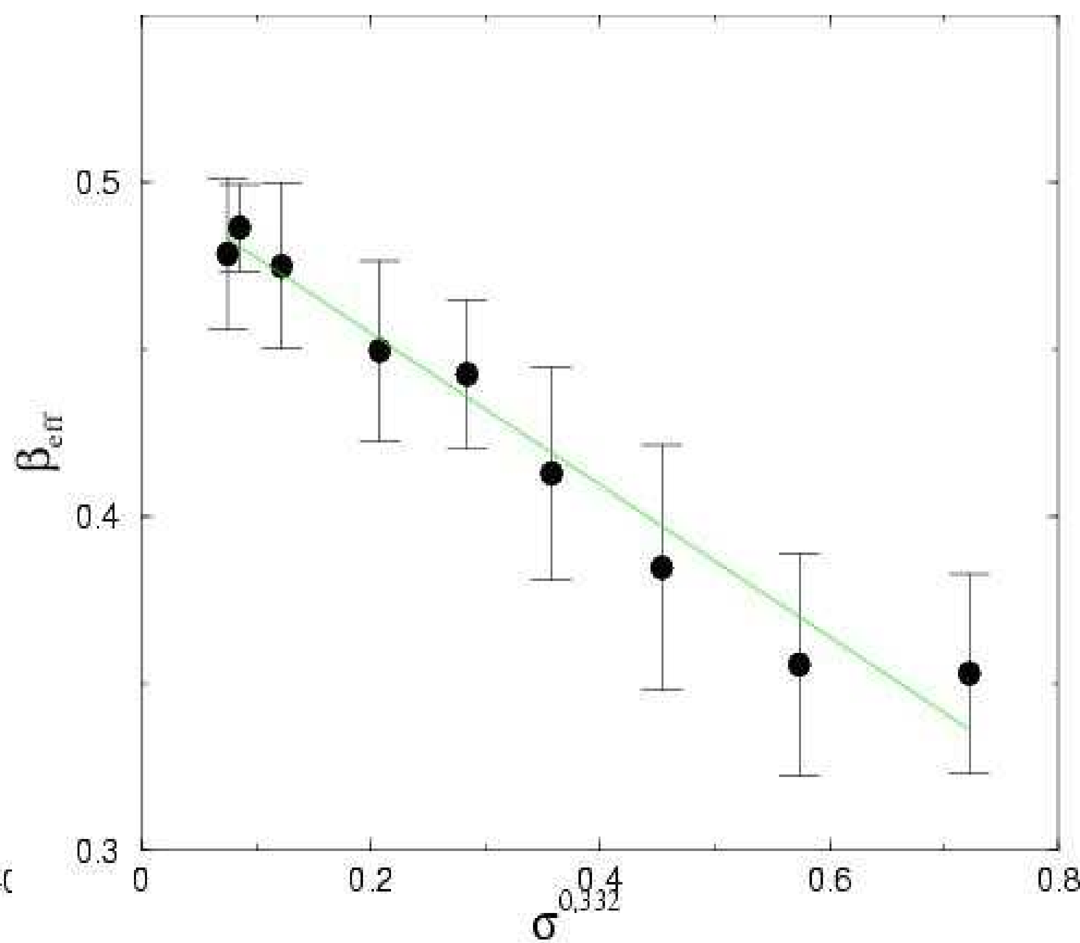




Simulation results for the 2A -> 4A, 4A -> 2A (site restricted) model in 1 dimension

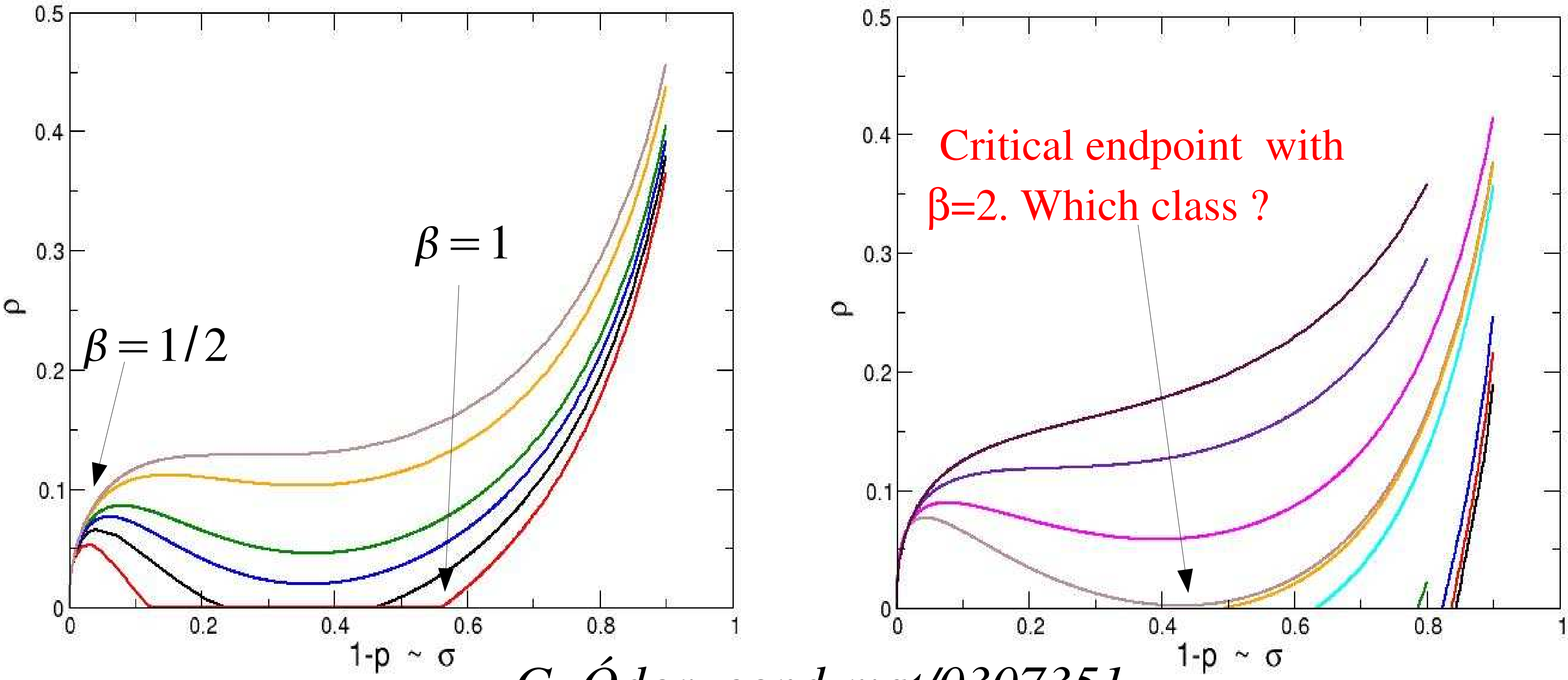
Mean-field type density decay and steady state behavior at D = 0.5 with α=1/3 and β=1/2 at zero branching rate. $d_c < 1$





Cluster mean-field results for the 2A -> 3A, 4A -> 0 model

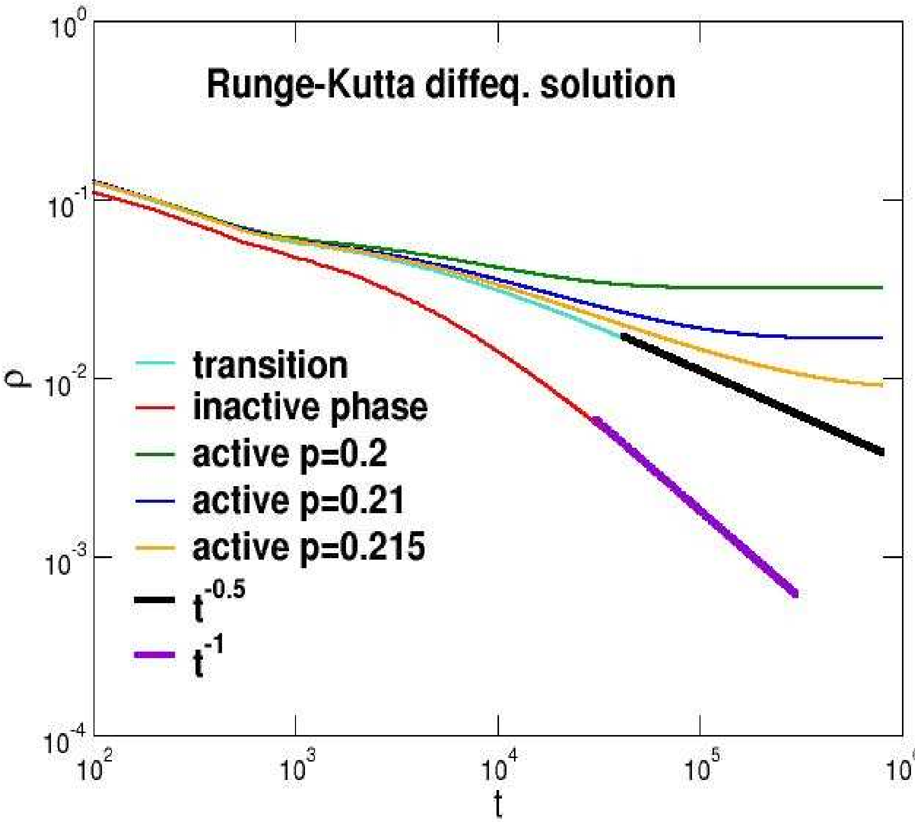
Steady state density for N = 2 and N = 4 level approximations.
Unexpected phase transition for σ > 0 with β=1, generated by diffusion.

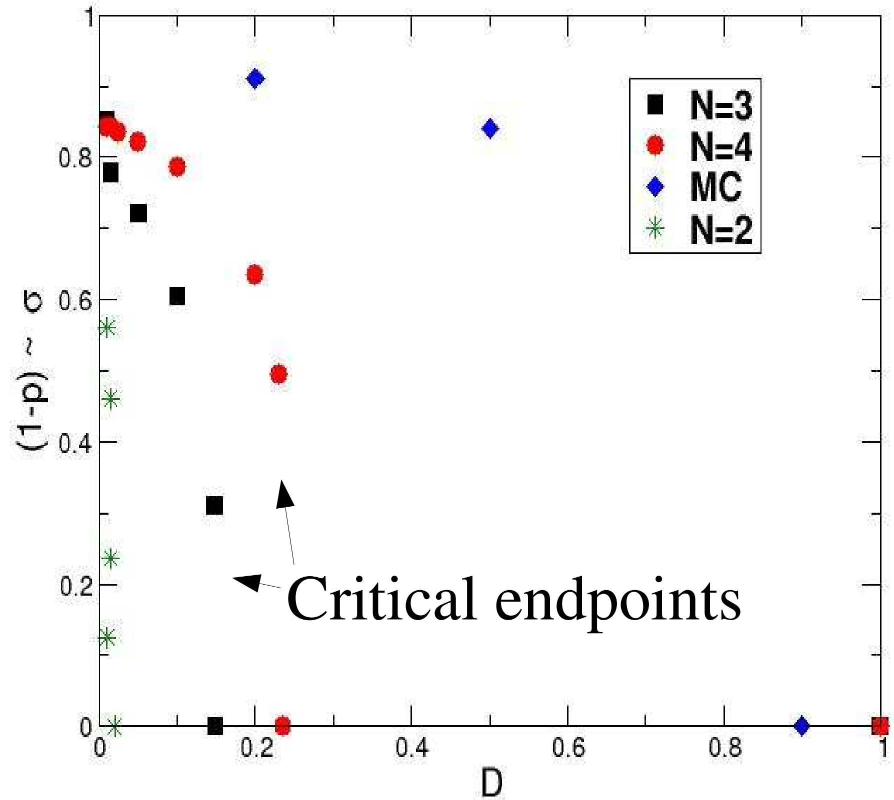


G. Ódor, cond-mat/0307351

Cluster mean-field results for the 2A -> 3A, 4A -> 0 model

Density decay in N = 3 level approximation: α= 0.5 (PCPD).
Phase diagram by N=2, 3, 4 mean-field and by Monte Carlo



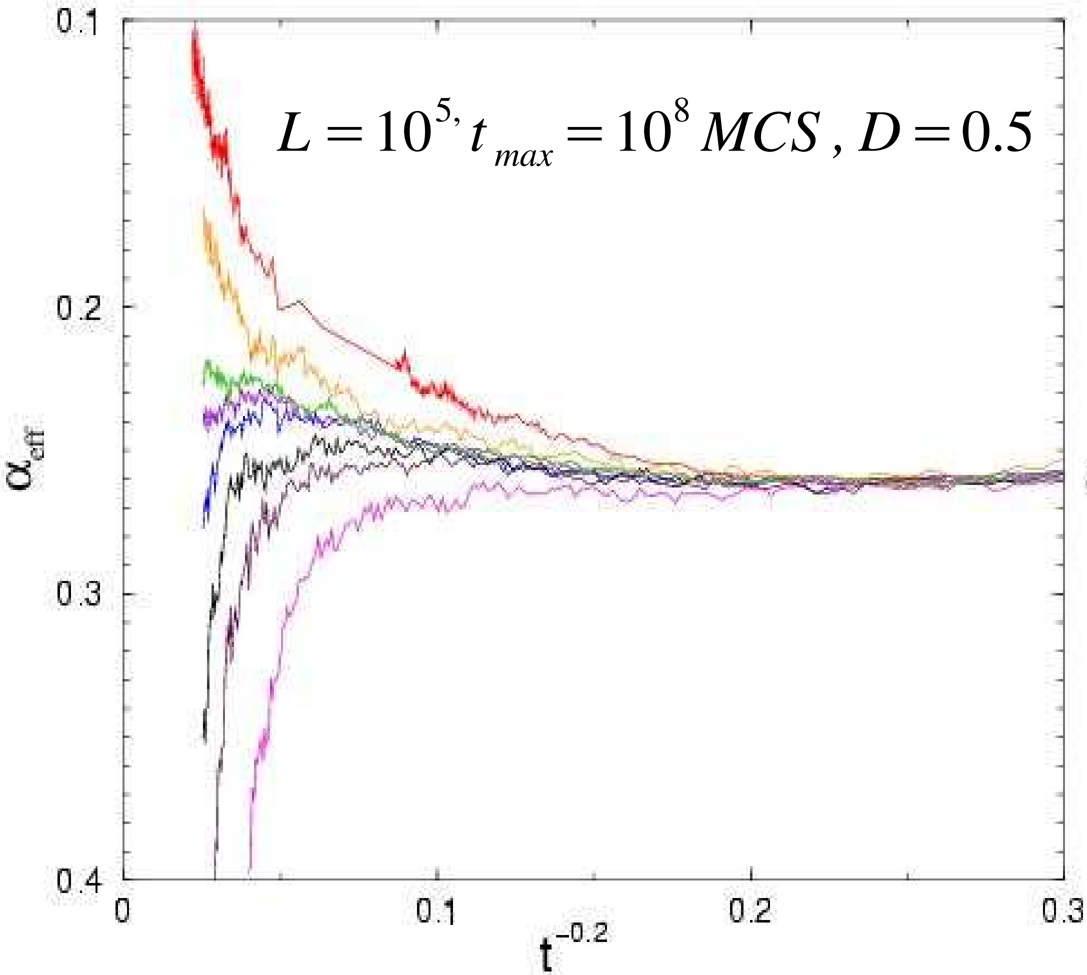


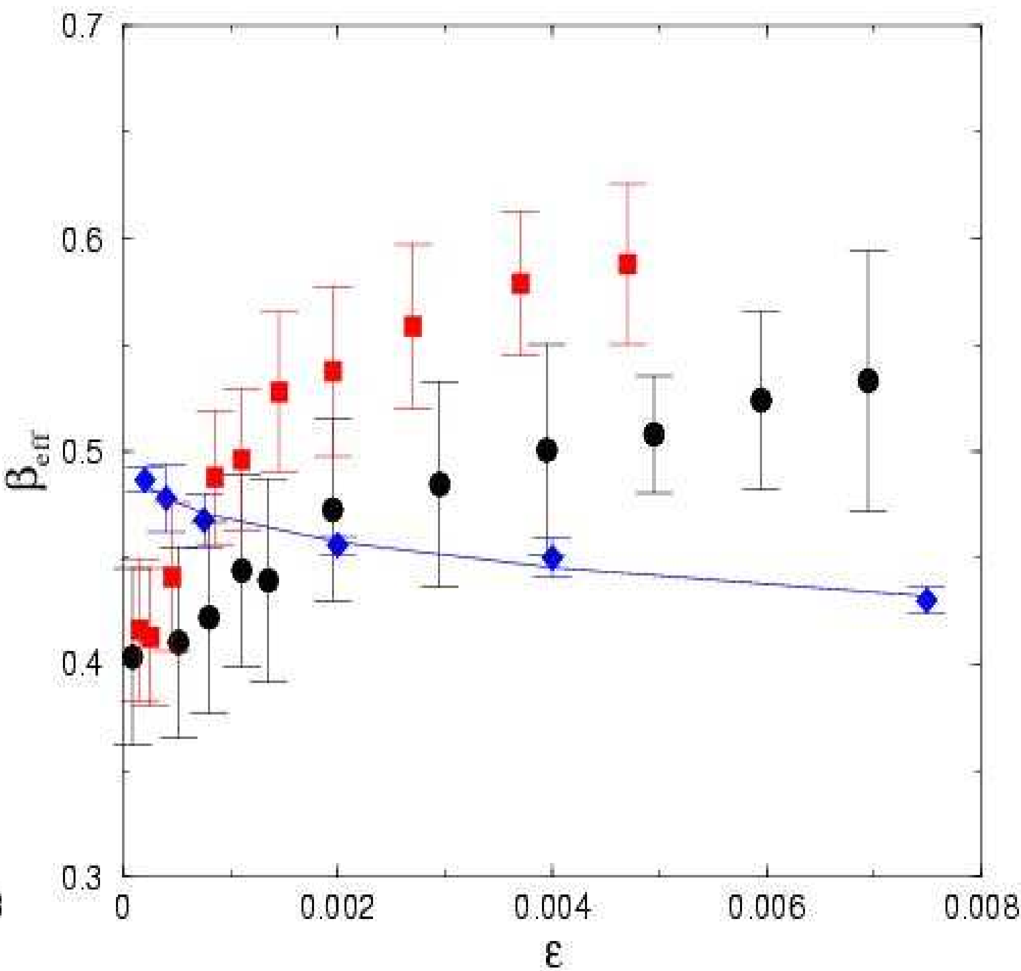
For small D and high σ the: 2A->3A->4A->0 process is relavant !

Simulation results for the 2A -> 3A, 4A -> 0 model in 1 dimension

Density decay local exponents with : $\alpha = 0.21(2)$ (~PCPD).

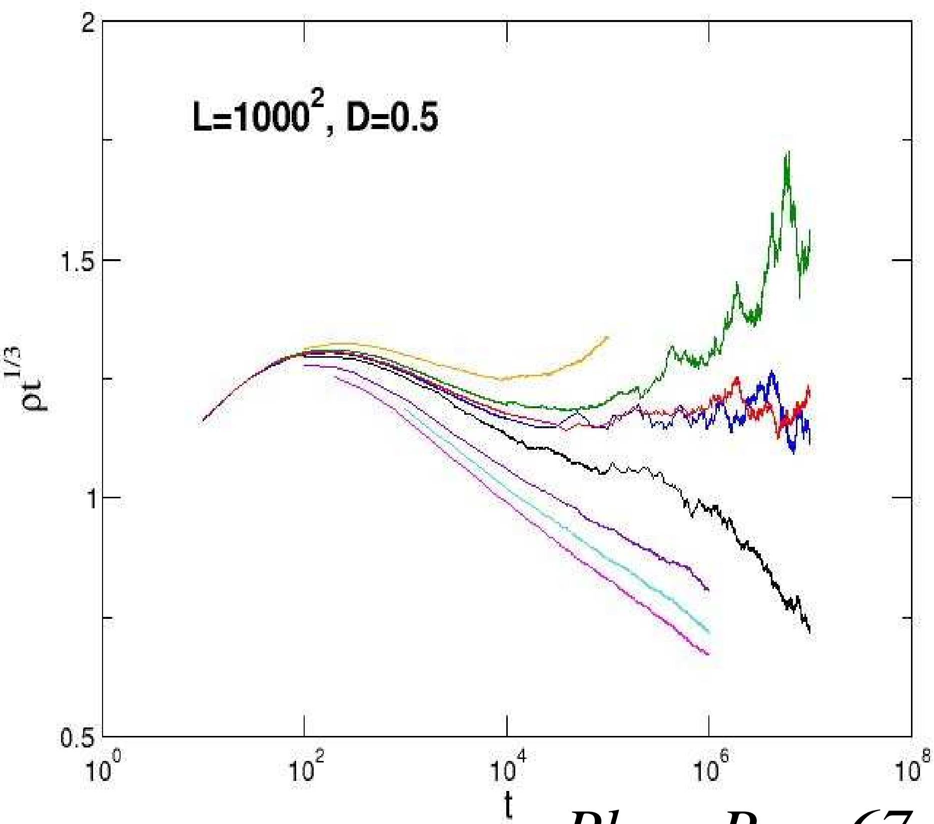
Steady state local exponents at D = 0.5, 0.2, with $\beta=0.40(2)$ (~PCPD), at D=0.9: mean-field $\beta=1/2$! 2A->0 process becomes irrelavant.

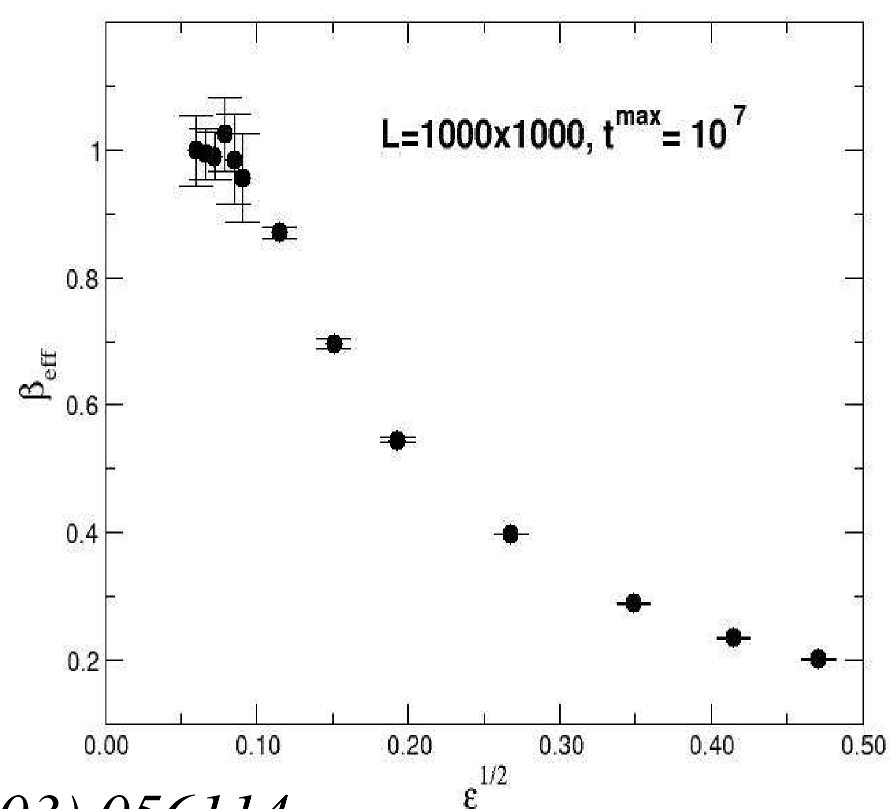




Simulation results for the 3A -> 4A, 3A -> 2A model in 2 dimensions

Density decay with mean-field value: $\alpha = 1/3$.
Effective β exponent with mean-field value (= 1).





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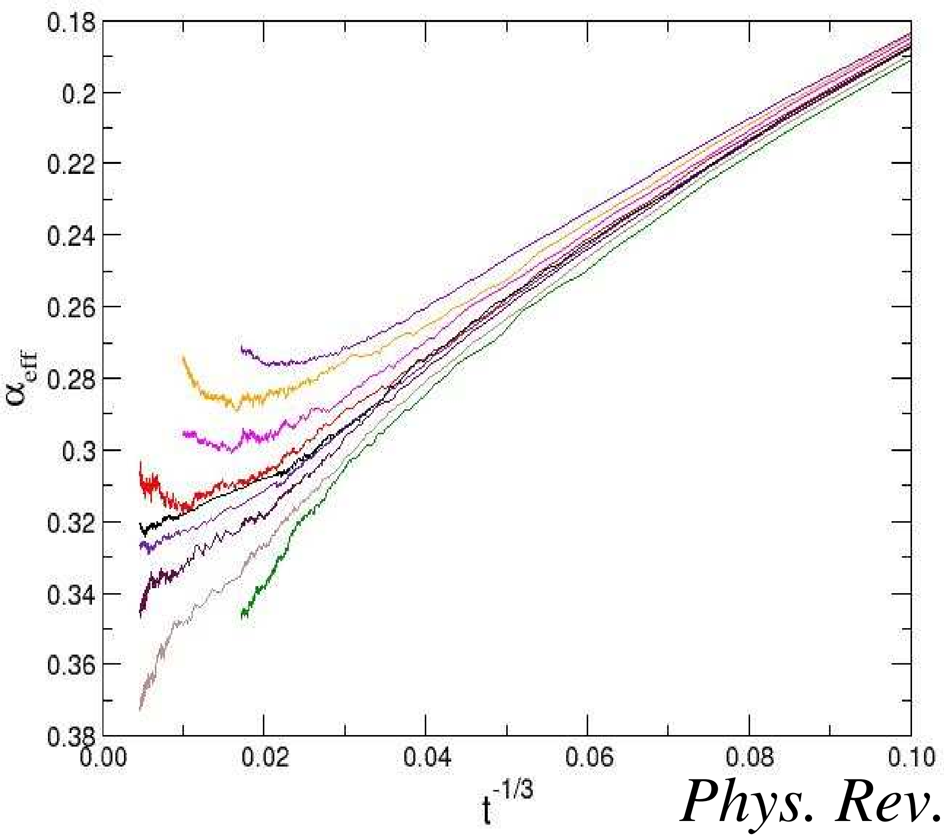
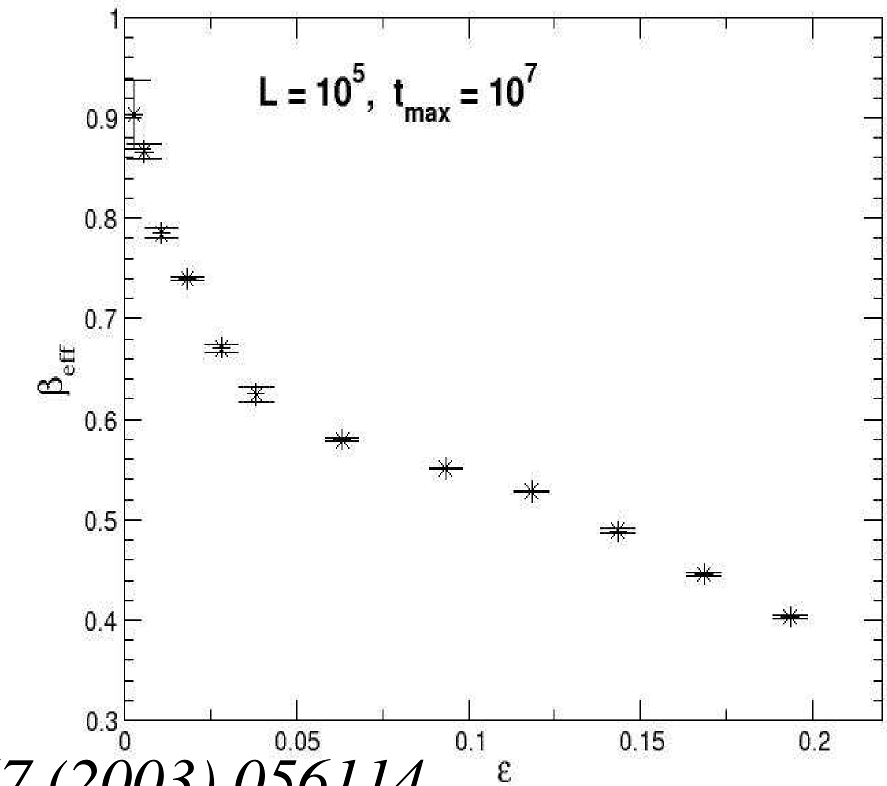
Simulation results for the 3A -> 4A, 3A -> 2A model in 1 dimension

Density decay effective exponent with mean-field value: $\alpha = 1/3$.

Effective β exponent with mean-field value. Small log corrections ?





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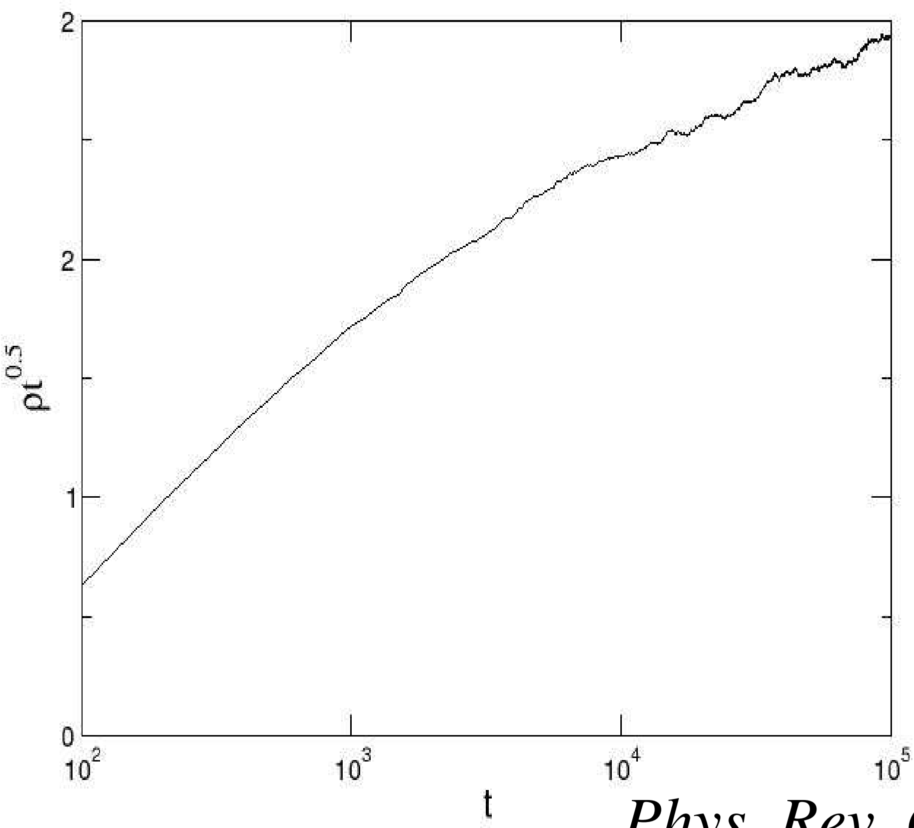
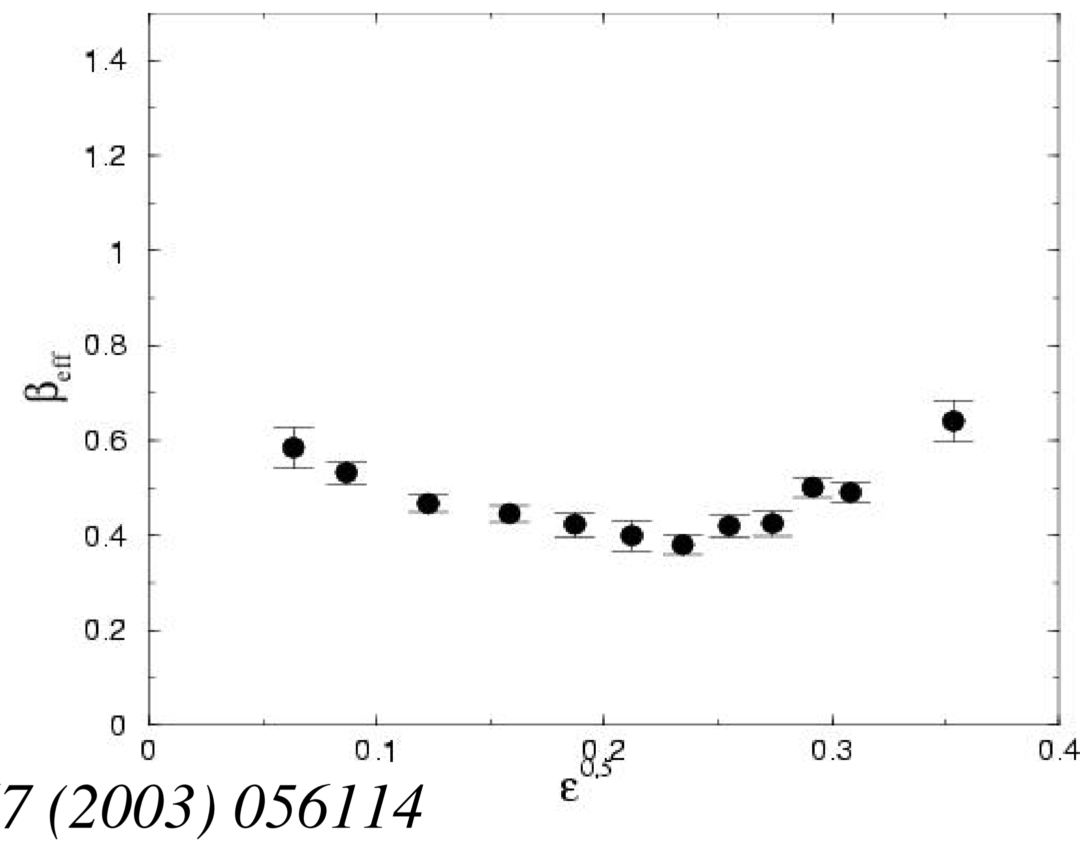
Simulation results for the 3A -> 6A, 3A -> 0 model in 1 dimension and D=0.2 diffusion

Phase transition at $\sigma = 0$. (a) Density decay: 3A->0 : $\rho \propto (\ln(t)/t)^{1/2}$
(b) Effective β exponent tends to: ~2/3 ? Which class ?





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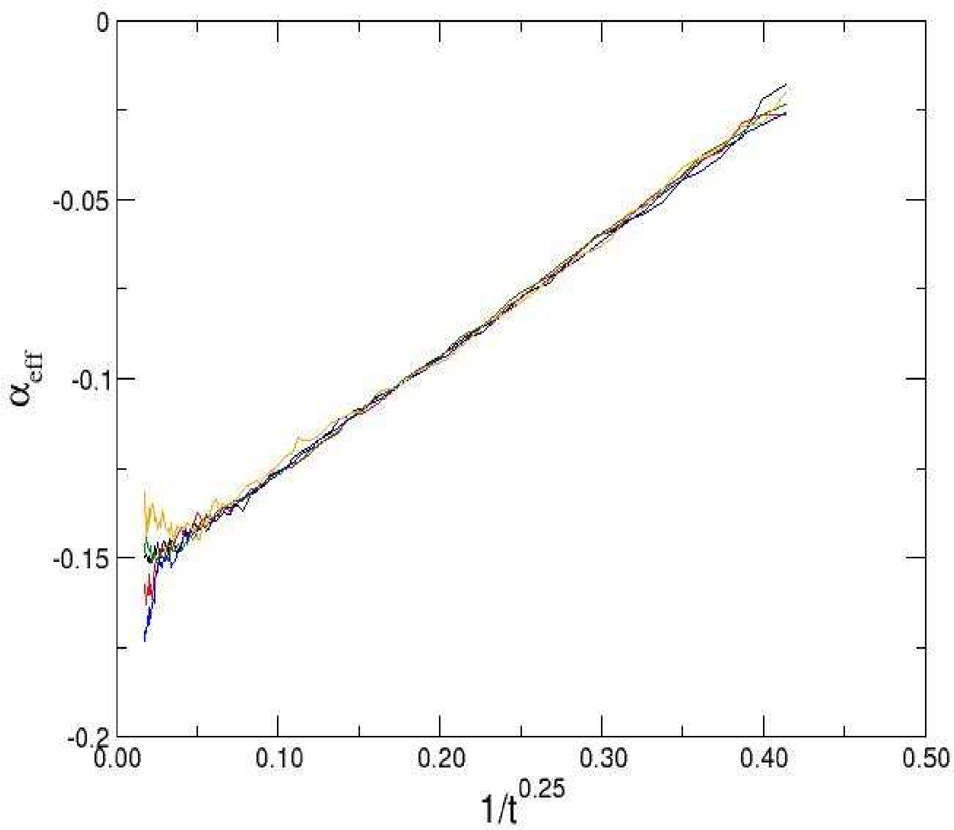
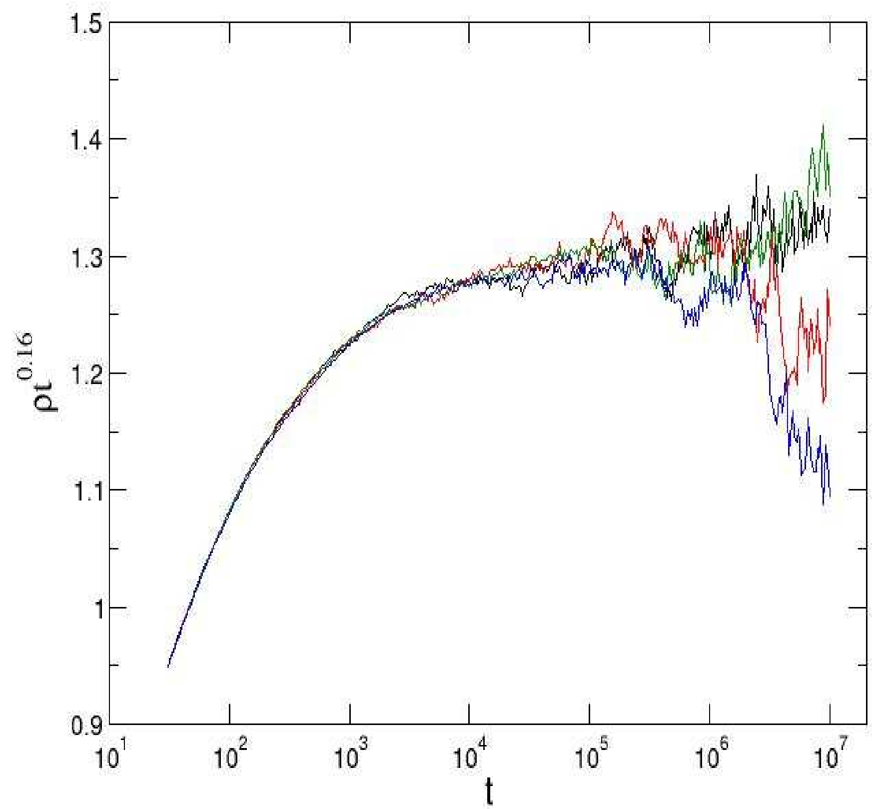
Simulation results for the 3A -> 5A, 2A -> 0 model in 1 dimension and D = 0.5 diffusion

Parity conserving triplet model. Mean-field: (valid for d ≥ 2) first order.
Density decay: DP class, with strong correction to scaling (α'=0.25).

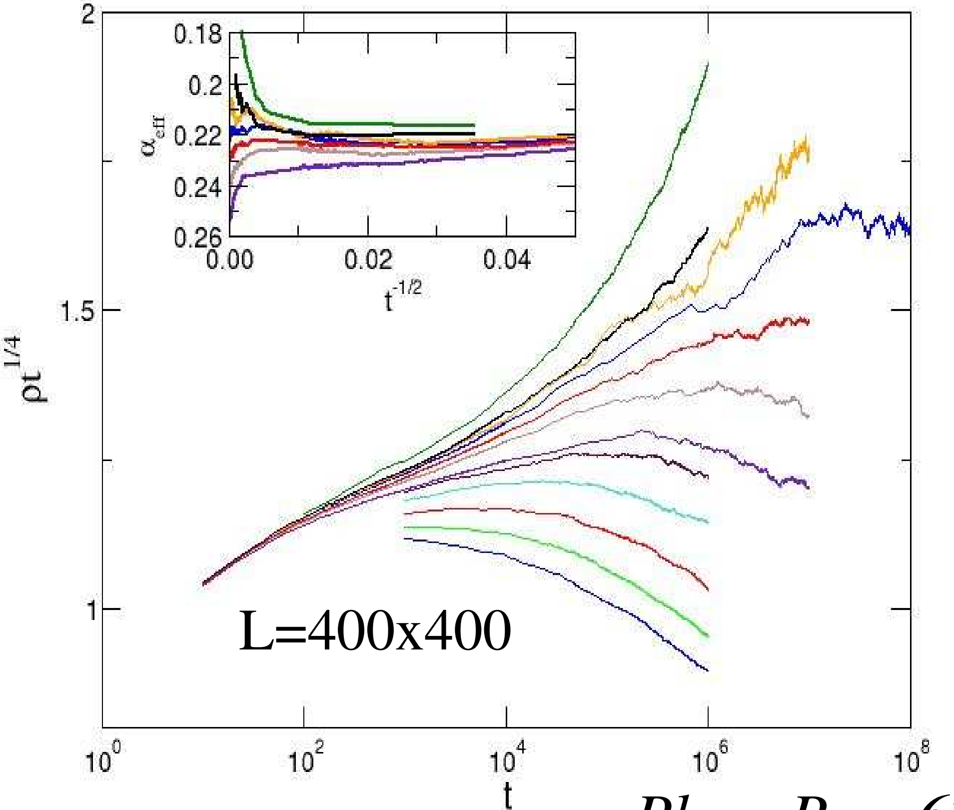


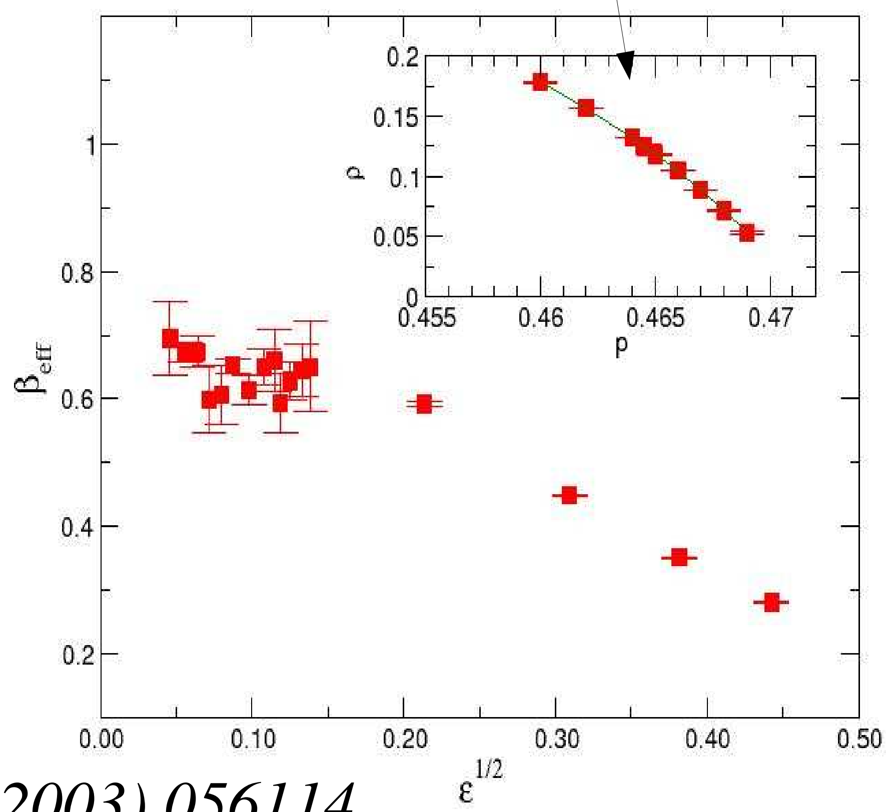


Simulation results for the 4A -> 5A, 4A -> 3A model in 2 dimensions and D = 0.5 diffusion

m=n type Quadruplet model. Mean-field: $\alpha = 1/4$, $\beta = 1$.
Density decay: log. Corrections, Steady state: logarithmic corrections

$$d_c = 2\,?$$





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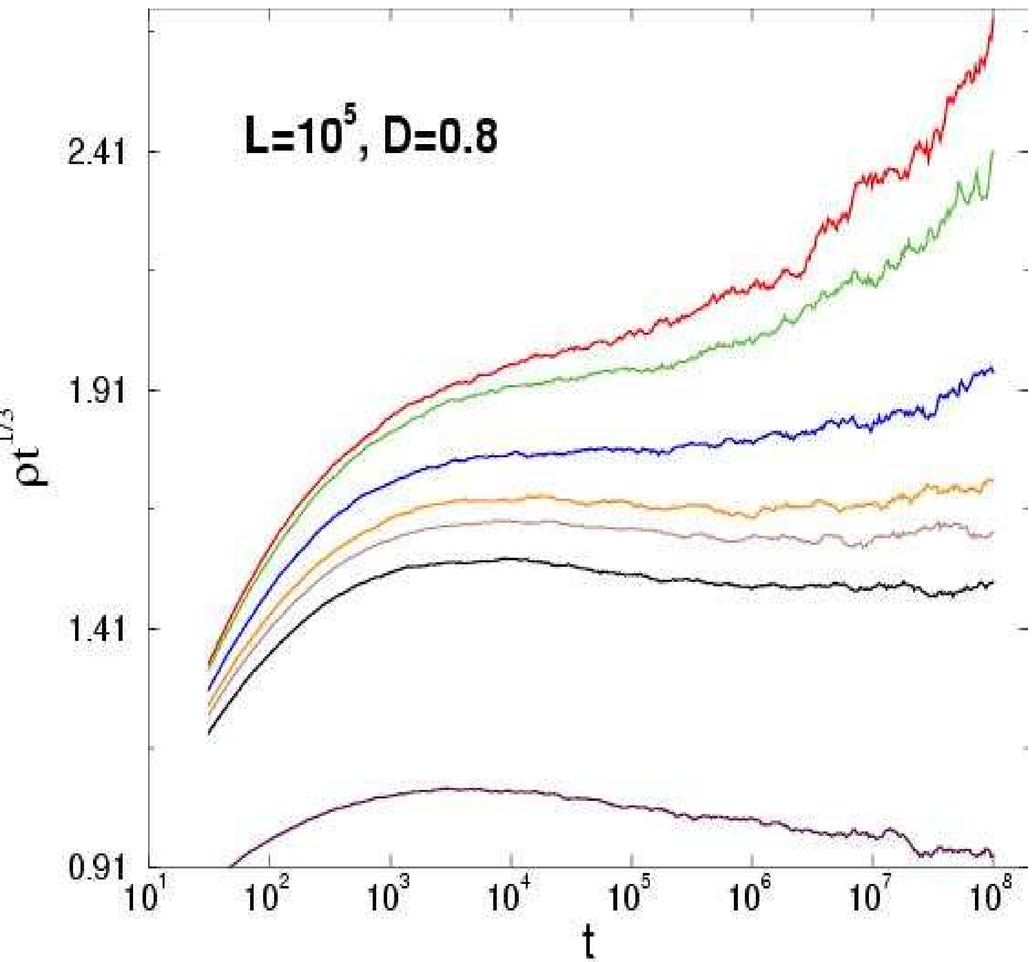
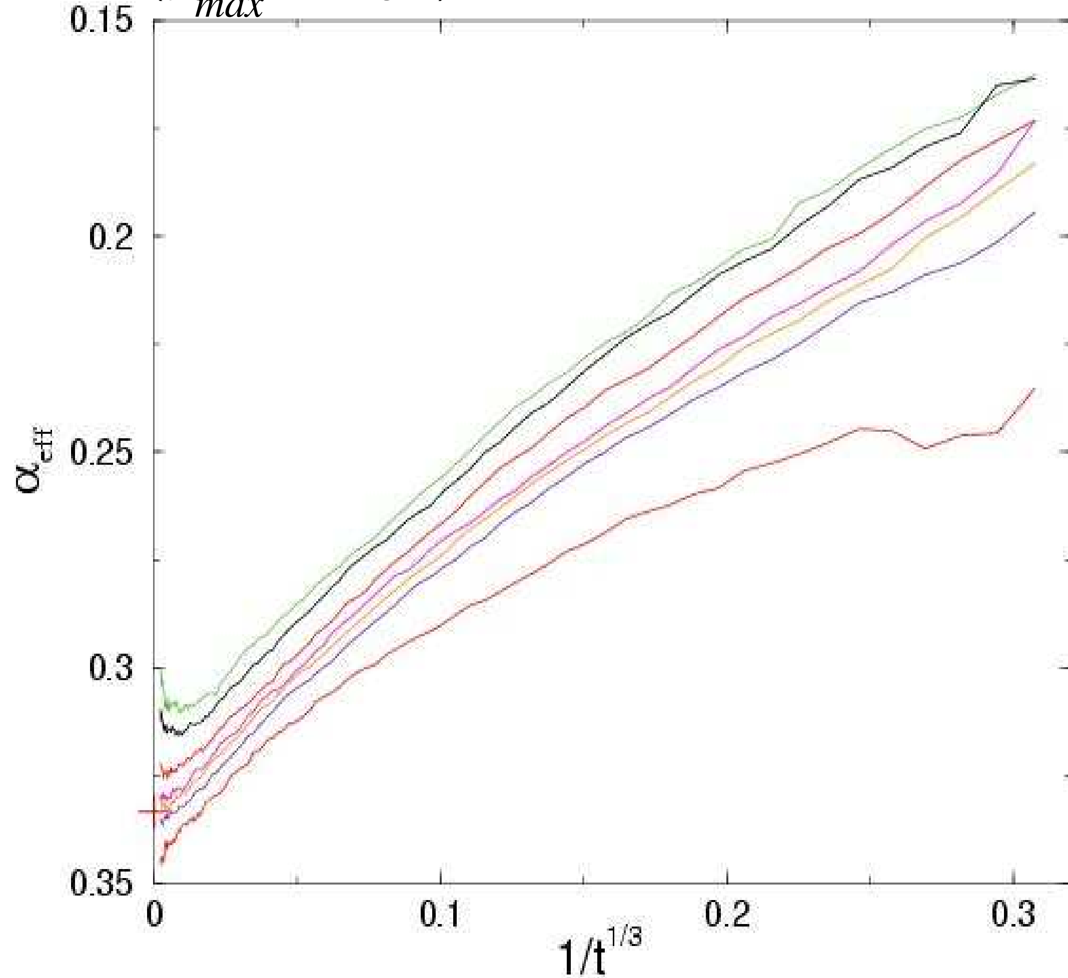
Simulation results for the 4A -> 5A, 4A -> 0 model in 1 dimension and D = 0.8 diffusion

m=n type Quadruplet model. Mean-field: $\alpha = 1/4$, $\beta = 1$.

In PRE 67 056114 (2003) non-mean-field ($t_{max} = 10^6$) . α=1/3 ?





Summary

- Mean-field classes of general, one-component RD systems are introduced.
- The importance of diffusion (not present in site mean-field) has been pointed out (PCPD; 2A->3A 4A->0; ....)
- For 2A->3A, 4A->0 model rich phase diagram.
- PCPD: in 2d: MF+log. Corrections, in 1d: novel+log. Corr.?
- Triplet : MF behavior, but for hybrid (3A->5A, 2A->0): DP.
- Quadruplet: in 2d: MF+log., in 1d: novel class ???