Home assignments

2.1. What is the suggestion of the minimax method for this zero-sum game?

$$G = \begin{pmatrix} (3,-3) & (1,-1) & (-2,2) \\ (2,-2) & (1,-1) & (4,-4) \\ (2,-2) & (-1,1) & (5,-5) \end{pmatrix}.$$

2.2. Find the pure NE of this bimatrix game:

$$G = \begin{pmatrix} (0,0) & (1,2) & (4,5) \\ (5,4) & (1,3) & (0,0) \\ (2,1) & (4,4) & (3,1) \end{pmatrix}.$$

2.3. Determine the mixed NE for a) the matching-pennies and b) the rock-paper-scissors game defined by:

a.) $G = \begin{pmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{pmatrix}$ and b.) $G = \begin{pmatrix} (0,0) & (1,-1) & (-1,1) \\ (-1,1) & (0,0) & (1,-1) \\ (1,-1) & (-1,1) & (0,0) \end{pmatrix}$.

2.4. In the Minority game each of N (odd) players has two options [to buy or to sell, to turn left or right on the way home, to go to the El Farol bar (Santa Fe) or to stay at home, etc.]. The winners will be those who belong to the minority and they receive +1, the others receive -1 payoff. How many pure Nash equilibria does this game have?

2.5. Draw the flow graph of the rock-paper-scissors game!

2.6. Describe 10 real life situations that resemble the prisoner's dilemma, donation, or public goods games!

3.1. How does the repeated PD game play out for the following strategy pairs: AllD-AllC, AllD-TFT, TFT-WSLS?

3.2. Show that WSLS can win in a population consisting of WSLS, AllC and TFT players! What happens if AllD players are present, too?

4.1. Determine the (p,q) strategies whose homogeneous population can be occupied by AllD! (The grey area on slide 8.)

4.2. Determine the boundary in the (p,q) strategy space where the evolution via weak mutations stops, that is, where the formulae on page 8 predict

$$\dot{p} = \left.\frac{\partial U(s,s')}{\partial p}\right|_{s=s'} = 0 \quad \text{and} \quad \dot{q} = \left.\frac{\partial U(s,s')}{\partial q}\right|_{s=s'} = 0 \quad !$$

Plot the results for the payoff parameters used by Axelrod (T=5, R=3, P=1, S=0)!

What is the optimal value of forgiveness (q_{opt}) in the limit $p \to 1$?

4.3. Determine the matrix **M** from the previous slide and evaluate the stationary solution!

6.1. Prove statements 6) and 7) from page 3!

6.2. The voluntary version of the Prisoner's dilemma game has three strategies labeled D, C, and L (L as in loner) and its payoff matrix can be given as:

$$\mathbf{A} = \mathbf{B} = \begin{pmatrix} 0 & T & \sigma \\ S & 1 & \sigma \\ \sigma & \sigma & \sigma \end{pmatrix}.$$

Is it a potential game?

6.3. What payoff matrix describes a symmetric 2×2 game extended by a third strategy that mixes its first and second strategies with probabilities q and $(1-q)$? Use the P=0, R=1, T, S parametrization! Prove that the resulting three-strategy game is a potential game!

7.1. Evaluate the coefficients $\alpha^{(n)}$ in the decomposition of the following antisymmetric 3x3 payoff matrix:

$$\mathbf{A}^{(\mathrm{as})} = \begin{pmatrix} 0 & a & -c \\ -a & 0 & b \\ c & -b & 0 \end{pmatrix} = \sum_{n=1}^{9} \alpha^{(n)} \mathbf{f}^{(n)},$$

where the 3×3 basis matrices $\mathbf{f}^{(n)}$ are those defined in the second example on page 3!

7.2. The payoff matrix of the (three-strategy) voluntary prisoner's dilemma is given by

$$\mathbf{A} = \begin{pmatrix} 0 & T & \sigma \\ S & 1 & \sigma \\ \sigma & \sigma & \sigma \end{pmatrix}$$

where $0 < \sigma < 1$ and the three strategies represent unconditional defectors, unconditional cooperators, and loners, respectively. Determine the strength of each elementary component!

Evaluate the value of σ for which a potential exists and determine the corresponding potential matrix!

7.3. The Ashkin–Teller model is a four-state version of the Ising model where the interaction between neighbouring players is defined by the payoff matrix:

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \alpha & \delta & \gamma \\ \gamma & \delta & \alpha & \beta \\ \delta & \gamma & \beta & \alpha \end{pmatrix}.$$

Determine the elementary coordination components of this matrix!

8.1. Show that mean-field approximation predicts a first-order phase transition for the four-state Potts model on a square lattice!

9.1. Evaluate the typical payoffs in the Nowak–May cellular automaton for z=8 when the domains of cooperators and defectors are separated by a horizontal boundary!

9.2. What are the payoffs in the previous problem if one of the players along the boundary reverses her strategy?

9.3. What are the strategy distributions at t+1 if the spatial distribution at t is the same as in 9.1. and 9.2. for b=1.3 and 1.7?

10.1. Show that cooperators become extinct on the one-dimensional chain with nearest neighbour interactions for the evolutionary rule defined on page 1 when the payoffs are $1<T<2$ and $S=0$!

10.2. Reproduce the phase diagrams plotted on page 13 by evaluating the potential matrix and determining its maximal entry (as a function of T and S for $Q=0$, 1/3, 1/2, and 1) that selects the preferred Nash equilibrium!

11.1. On a one-dimensional lattice for Q=2, the three-site cluster configuration probabilities satisfy the following relations:

$$p_3(1,0,0) = p_3(0,0,1) \text{ and } p_3(1,1,0) = p_3(0,1,1),$$

whereas for the four-site configuration probabilities

$$p_4(1,1,0,0) \neq p_4(0,0,1,1) \text{ but } p_4(1,0,0,0) = p_4(0,0,0,1).$$

What is the common feature of configurations that can exhibit symmetry breaking for n>4?

11.2. Show that on a one-dimensional lattice for Q=3 the compatibility conditions allow the following relations:

$$p_2(1,0) - p_2(0,1) = p_2(2,1) - p_2(1,2) = p_2(0,2) - p_2(2,0).$$

What is the main feature of the patterns that can be described by the following configuration probabilities:

$$p_2(0,0) = p_2(1,1) = p_2(2,2) = a$$

$$p_2(1,0) = p_2(2,1) = p_2(0,2) = \frac{1}{3} - a$$

$$p_2(0,1) = p_2(1,2) = p_2(2,0) = 0$$

11.3. Find a suitable parametrization for the cluster configuration probabilities on a 2×2 cluster of sites on a square lattice for Q=2, if the system exhibits all the possible symmetries (reflection and rotation)! How many independent parameters does it have?

11.4. Determine the number of parameters we need to introduce at the level of triangular cluster approximation on the kagome and triangular lattices for Q=2!

11.5. Evaluate the specific entropy

$$S = -\frac{1}{N}\sum_{\{s_x\}} p_N(\{s_x\})\ln\left[p_N(\{s_x\})\right]$$

in a Q-state one-dimensional system in the $N\to\infty$ limit when

(1) $\quad p_N(\{s_x\}) = \prod_{x=1}^{N} p_1(s_x),$

(2) $\quad p_N(\{s_x\}) = p_1(s_1)\prod_{x=1}^{N-1}\frac{p_2(s_x, s_{x+1})}{p_1(s_x)}.$

12.1. Determine the periodic time of oscillations in the population dynamics of the rock–paper–scissors game in the $\varepsilon \to 0$ limit, if $\rho_1(t=0)=1/3+2\varepsilon$ and $\rho_2(t=0)=\rho_3(t=0)=1/3-\varepsilon$! (Advice: Use the formalism of complex functions, that is, $\delta\rho_k(t)=\rho_k(t)-1/3=\varepsilon \mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}2\pi(k-1)/3}$ +c.c.).

13.1. The six-species model introduced on page 5 is defined by a food web that is a directed graph. A directed graph can be characterized by its adjacency matrix **A** whose elements A_{ij} are 0 if species i and j are neutral (not linked); A_{ij}=1 if there is a link directed from i to j; and A_{ij}=–1 if there is a link from j to i. Determine the adjacency matrix for this system! (Note: This adjacency matrix can be considered as the payoff matrix for this six-strategy game.)

13.2. What are the conserved quantities in the population dynamics of the model mentioned above in 13.1.?