Extended spatial evolutionary games Lecture 10

Traditional model: *N* players are located at the sites of a lattice or graph.

Each player *x* follows one of the pure strategies denoted by

$$s_x = D = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{(defector)} \quad \text{or} \quad C = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{(cooperator)}$$

Player *x* receives a payoff from games played with her neighbours at (x+δ)

$$U_x = \sum_{\delta} s_x^{+} \mathbf{A} s_{x+\delta}\,, \qquad \mathbf{A} = \begin{pmatrix} 0 & b \\ 0 & 1 \end{pmatrix},$$

The strategy of a randomly selected nearest neighbour is adopted with probability

$$w(s_x \to s_y) = \frac{1}{1 + \exp[(U_x - U_y)/K]}$$

Random initial state $\rightarrow$ stationary state (ρ: frequency of strategy *C*)

Phase diagrams on different networks (z=4)

Simulation: □ : square lattice

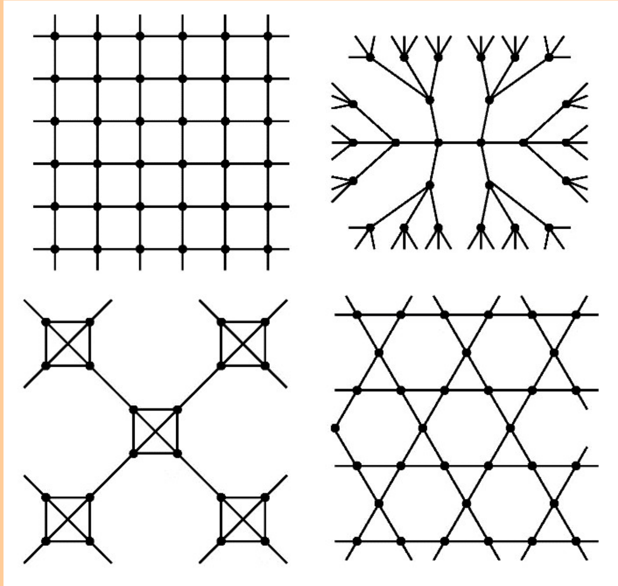
Δ : kagome lattice

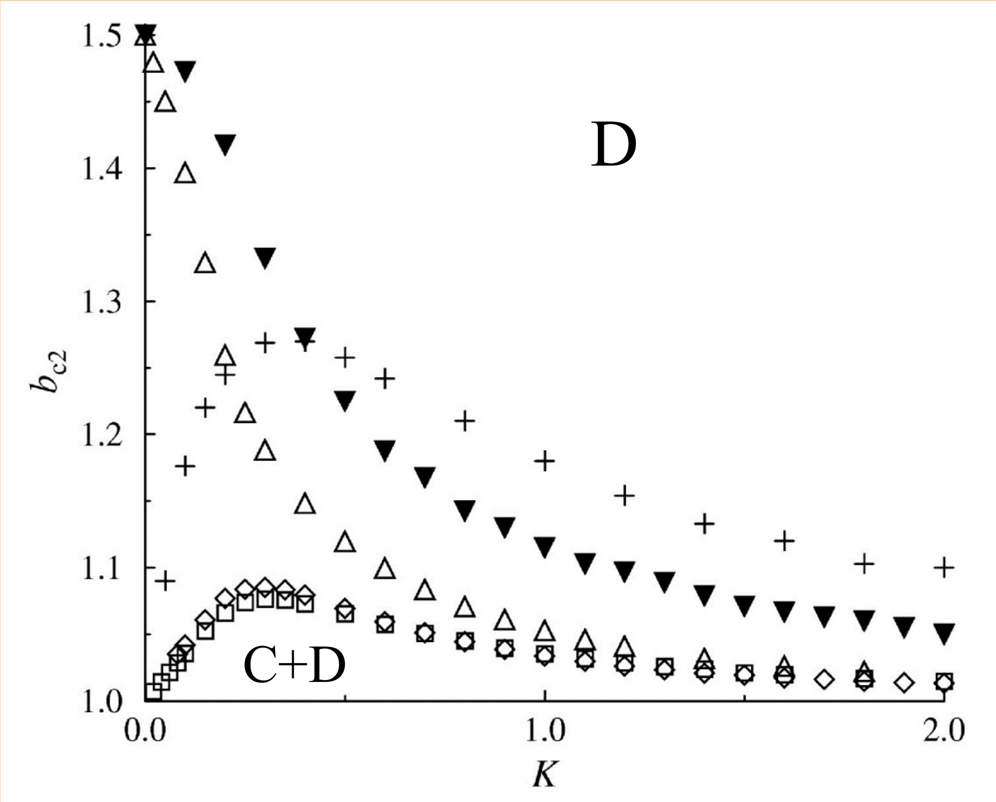
+ : Bethe lattice

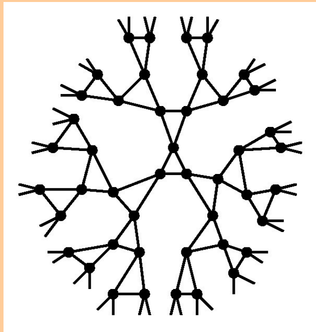
◊ : square lattice of 4-site cliques

▼ : regular graph of overlapping triangles

Noise-dependence: homogeneous D state if $b>b_{c2}$







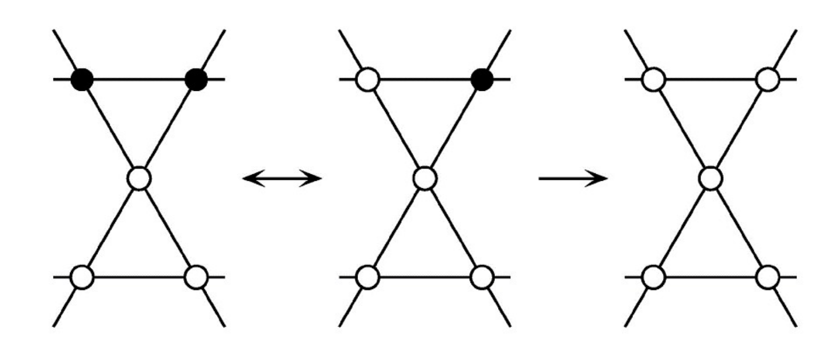
RG3

Two types of phase diagrams

Spatial structures do not support C for low noises(?)

Spreading of *C* through overlapping triangles

C (○) triplet in a sea of *Ds* (●), in the limit $K \rightarrow 0$



Payoff for a *C* within the triangle: $U_C=2$

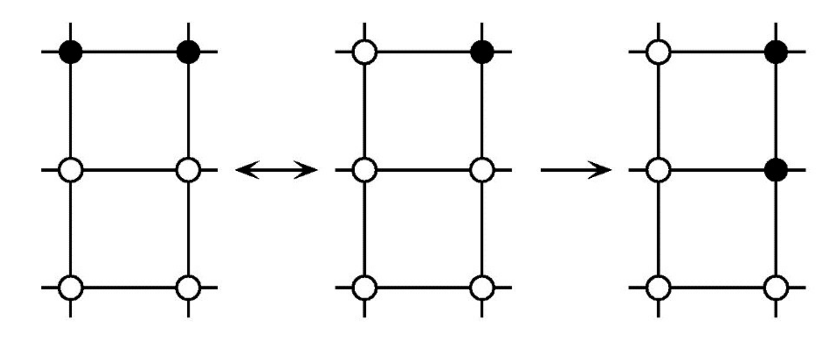
a neighbouring *D*: $U_D=b$

a *D* within the sea: $U_D=0$

The *C* triangle is stable and catalyzes the spreading of *C* via the growth of branches of triangles.

The growth of branches is blocked when two branches collide, separated by a single *D*.
For RG3 this is not possible, so it maintains a higher level of cooperation.

Counterexample:



the same mechanism on the square lattice:
Payoff for a *C* within the square: $U_C=2$
for a neighbouring *D*: $U_D=b$
for a *D* within the sea: $U_D=0$
The four-site *C* cluster disappears

MC simulations: spreading of *C* through overlapping triangles supports the maintenance of cooperation for structures in which overlapping triangles span the whole lattice (e.g. triangular lattice, fcc, bcc, kagome, RG3, etc.).

Inhomogeneous neighbourhood Santos and Pacheco, *PRL* **95** (2005) 098104

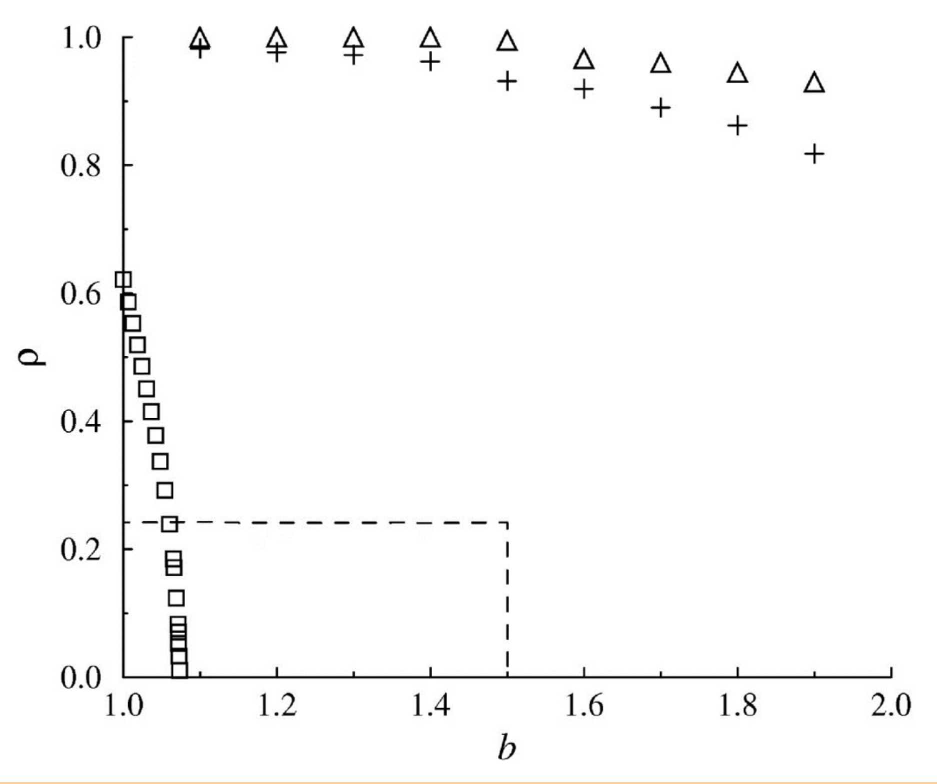
In real social systems, the number of neighbours (z) varies widely.

models: diluted lattice

small-word networks (Watts–Strogatz)

scale-free networks [$f(z) \sim z^{-3}$] (e.g., BA and DMS models)

Monte Carlo results on different networks: (K is optimum or low)



Dorogovtsev–Mendes–Samukhin model: Δ

Barabási–Albert model: +

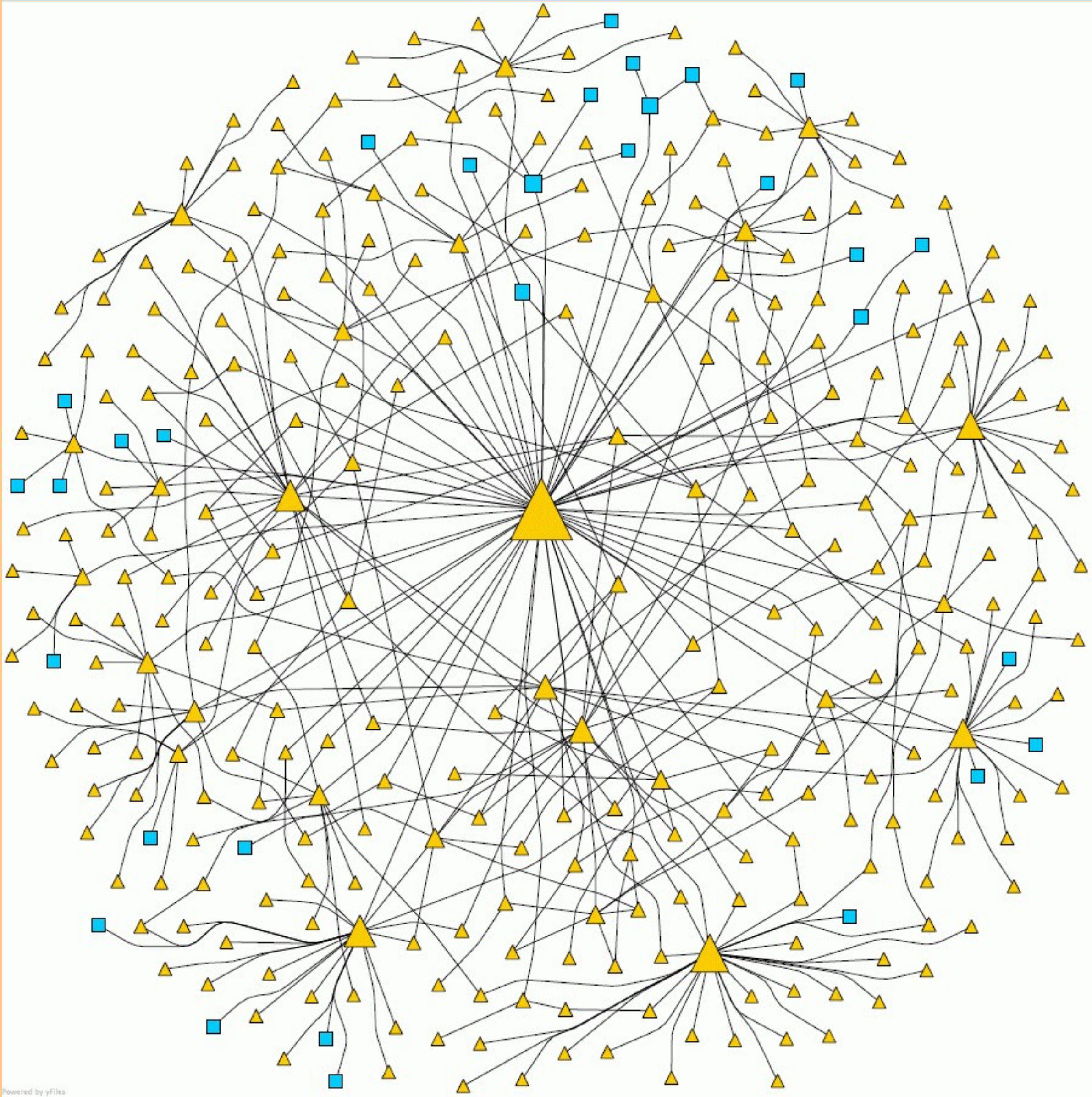
kagome lattice (K=0): - - - - -

Square lattice: □ (optimum K)

Inhomogeneous degree distribution supports cooperation!

Distribution of *C* (▲) and *D* (■) strategies on a scale-free random network

(Luthi et al., *Biosystems* **96** (2009) 213)

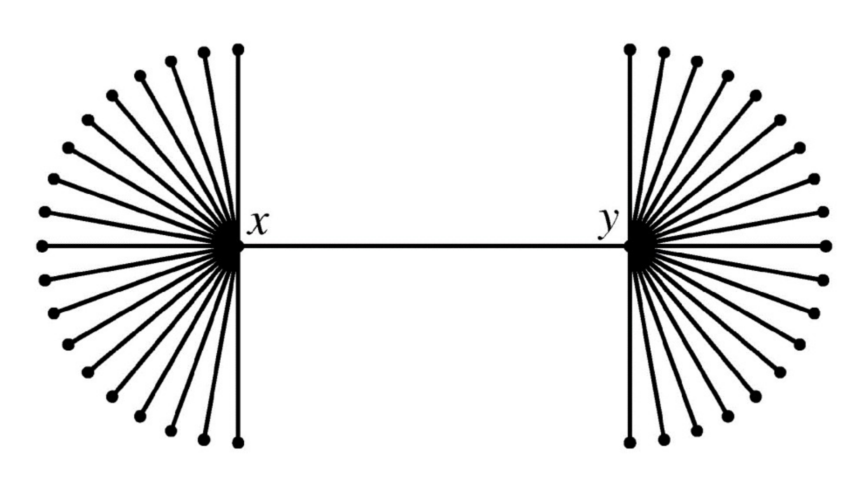


The size of a symbol is proportional to its degree

Another mechanism supporting cooperation:

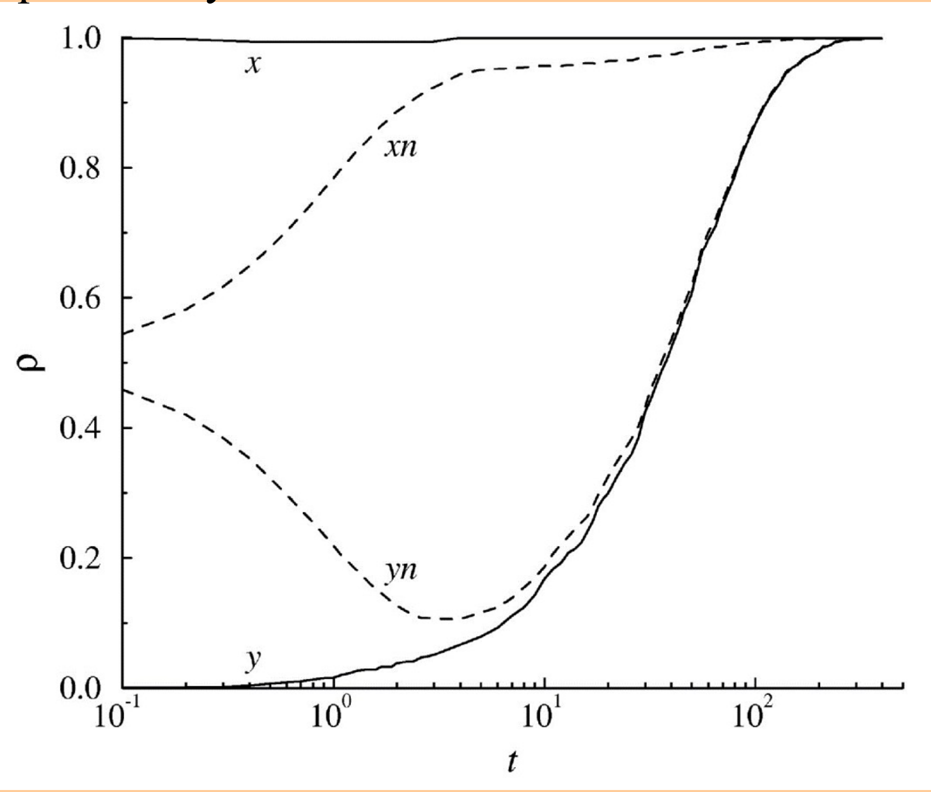
Subsystem with two linked hubs,
both with many neighbours

$$s_x = C \quad \text{and} \quad s_y = D \quad \text{at} \quad t = 0$$



Numerical simulation

The effect of the omitted players is taken into consideration via random strategy adoption with a probability of P=0.1.



Random initial distribution, n_x=n_y=49
Average over 1000 runs

Initially: U_x>U_{xn} and U_y>U_{yn}

Consequences: s_{xn}→C and s_{yn}→D, subsequently s_x=C becomes the most successful (hub) player to be followed by others.

Coevolutionary games:

Some or all ingredients of the system are allowed to evolve.

Examples: both the strategy distribution and the connectivity structure can evolve

players can choose another coplayer or move away

players can join or leave a community

personal features can be varied and/or inherited

e.g. reputation, age, sympathy, etc.

additional features can vary too (payoffs, rules, set of strategies, etc.)

The complexity (or the number of parameters) increases,

the system's behaviour becomes richer.

The mathematical description may be simplified by considering the personal features as adoptable strategies.

On the other hand, evolution can even be applied to the model parameters to select those that are relevant.

Influential players

The same spatial evolutionary PD game as before, but two types of players (*A* and *B*) are distinguished.

Their features: strong or weak capability of convincing others.

The probability of strategy adoption from *y* to *x*

$$w(s_x \to s_y) = W_y \frac{1}{1+\exp[(U_x - U_y)/K]}$$

$$\text{where} \quad W_y = \begin{cases} 1 & \text{if} \quad n_y = A \\ w & \text{if} \quad n_y = B \end{cases} \quad \text{and} \quad w < 1$$

v portion of players belong to type *A* (called influential players).
The initial spatial distribution of *A*- and *B*-type players remains unchanged during the evolution.

The role of the influential players resembles the role of hubs in the previous model, and consequently, we expect similar improvement in the level of cooperation.

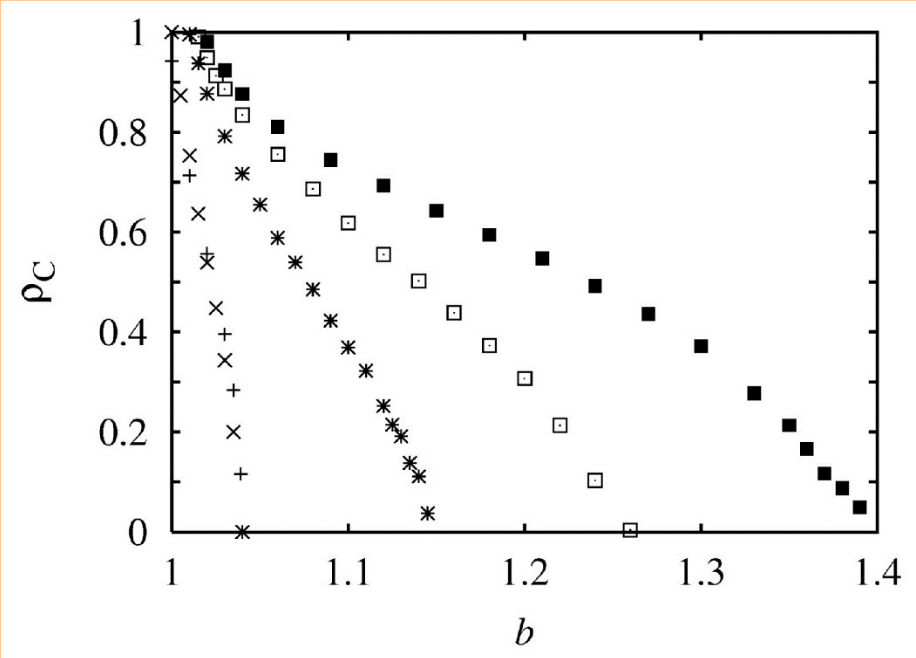
Simulations on square lattice for *z*=24

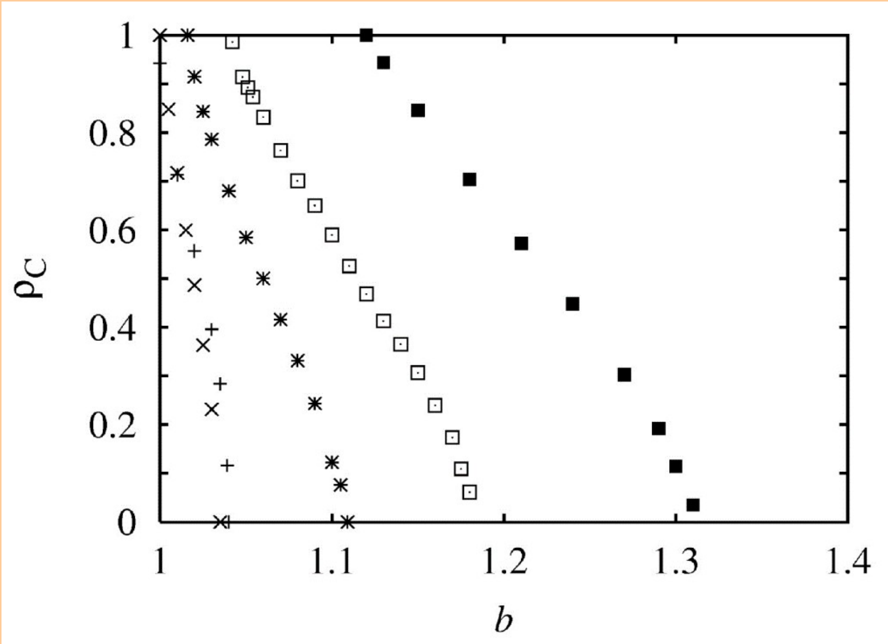
For low *v* values, the influential players cannot pass their strategy to others.

MC data for *v*=0.02, if *w*=1.0, 0.2, 0.05, 0.02, and 0.005

significant increase in ρ_C when *v*=0.1

or when we allow the players to move *A* players can switch places with their neighbours





Simulation for fixed players

Simulation with moving *A*s

Fraternal behaviour in evolutionary social dilemmas

N players are located at the sites of a square lattice (with periodic boundary conditions)

Players can choose one of the pure strategies

$$s_x = D = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{(defector)} \quad \text{or} \quad C = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{(cooperator)}$$

Each player x plays games with her neighbours ($x+\delta$) to collect income.

But now they also take their coplayers' income into consideration as

$$U_x = \sum_\delta \left[(1-Q) s_x^+ \mathbf{A} s_{x+\delta} + Q s_{x+\delta}^+ \mathbf{A} s_x \right] \quad \text{where} \quad s_x^+ \mathbf{A} s_y = (s_{x1} \quad s_{x2}) \begin{pmatrix} 0 & T \\ S & 1 \end{pmatrix} \begin{pmatrix} s_{y1} \\ s_{y2} \end{pmatrix}$$

Logit rule: player x modifies her strategy to s'_x with probability

$$w(s_x \to s'_x) = \frac{1}{1 + \exp[(U_x - U'_x)/K]}$$

The effective payoff matrix:

$$\mathbf{A} \to \mathbf{A}_{\text{eff}} = \begin{pmatrix} 0 & (1-Q)T + QS \\ (1-Q)S + QT & 1 \end{pmatrix}$$

K: noise

Q: measure of altruism

Q=0: selfish

Q=1/2: fraternal

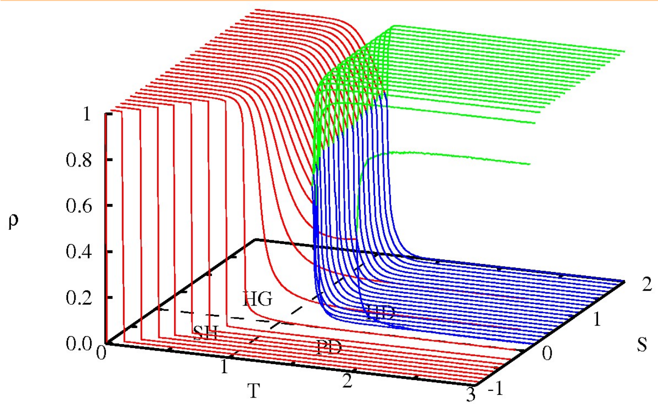
Q=1: lover

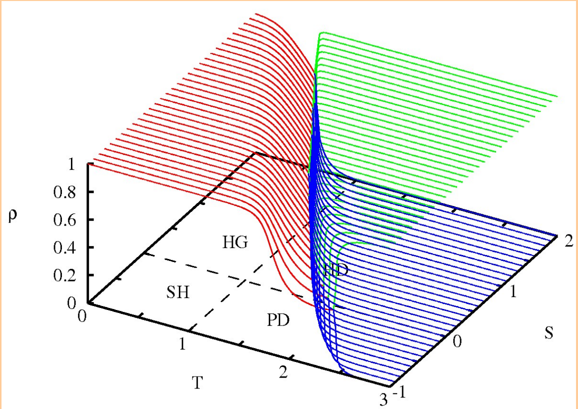
MC simulation at *K*=0.25 and *z*=4 for logit rule

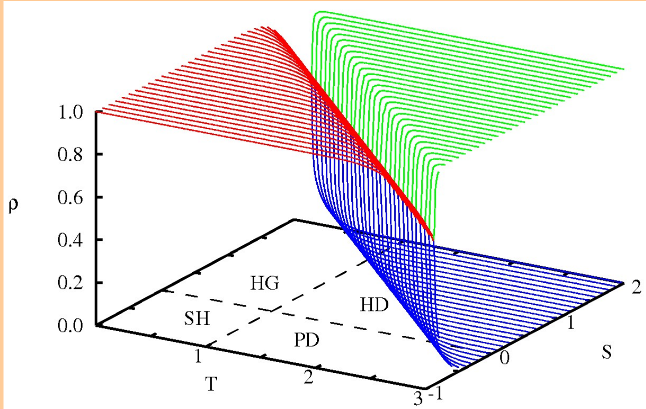
The lattice is divided into two sublattices *A* and *B*.

ρ_A (blue lines) and ρ_B (green lines) denote the fraction of Cs in the corresponding sublattices.

Q=0	*Q*=1/3	*Q*=1/2
selfish	„big brother"	fraternal







Sublattice ordering (separation of roles):

red lines: the sublattices are equivalent

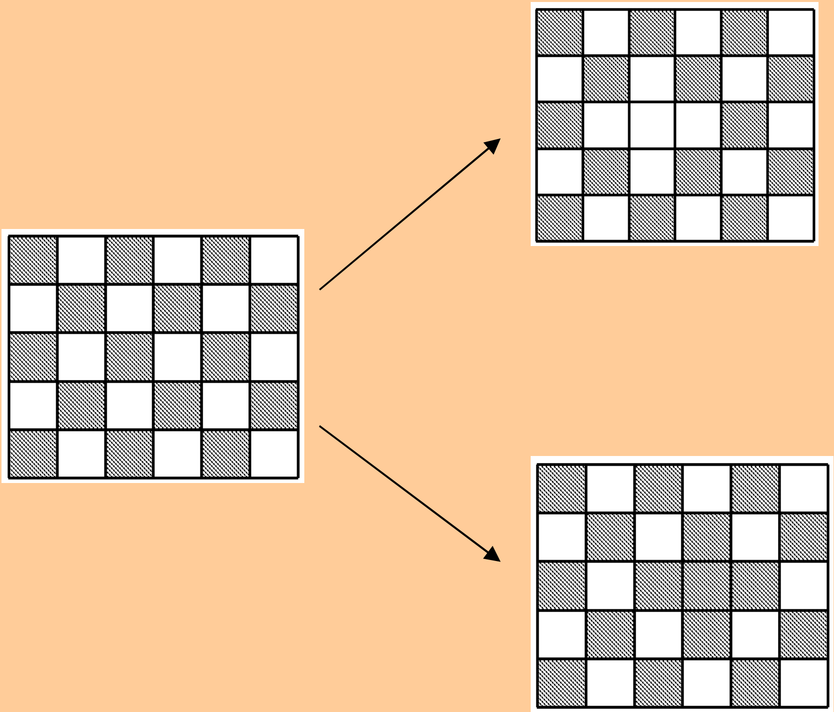
Q=1/2: social optimum is available in the $K \rightarrow 0$ limit (no dilemma)

The width of the transition regions is proportional to *K*.

The phase boundaries can be evaluated by stability analysis in the $K \rightarrow 0$ limit.

Stability of the sublattice ordered strategy arrangement

Creation of a point defect: (P=0, R=1)



$D \rightarrow C$ replacement is favoured if

$U_x=4[(1-Q)T+QS] < U_x'=4$,

that is, if $QS < 1-(1-Q)T$.

chessboard → homogeneous C

$C \rightarrow D$ change is favoured if

$U_x=4[(1-Q)S+QT] < U_x'=0$,

that is, if $(1-Q)S < -QT$.

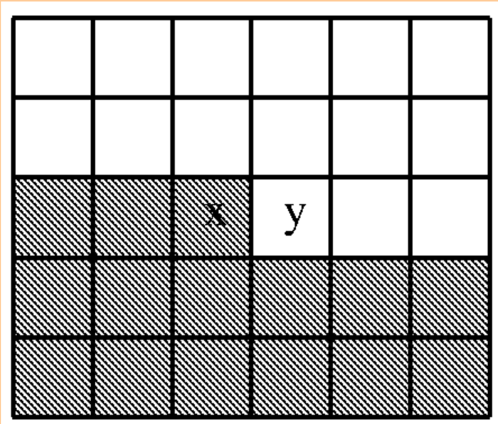
chessboard → homogeneous D

These point defects are generated independently of each other. As a consequence, the chessboard state can be transformed into the homogeneous C or D state (or vice versa).

Phase boundaries: $Q(S-1) = -(1-Q)(T-1)$, or $(1-Q)S = -QT$

Direction of invasion along the interface separating *C* and *D* domains (SH region)

The motion of this step determines the direction of invasion between
C (white) and *D* (grey) domains



$U_x = 2[T(1-Q) + QS]$, $R=1$, $P=0$,

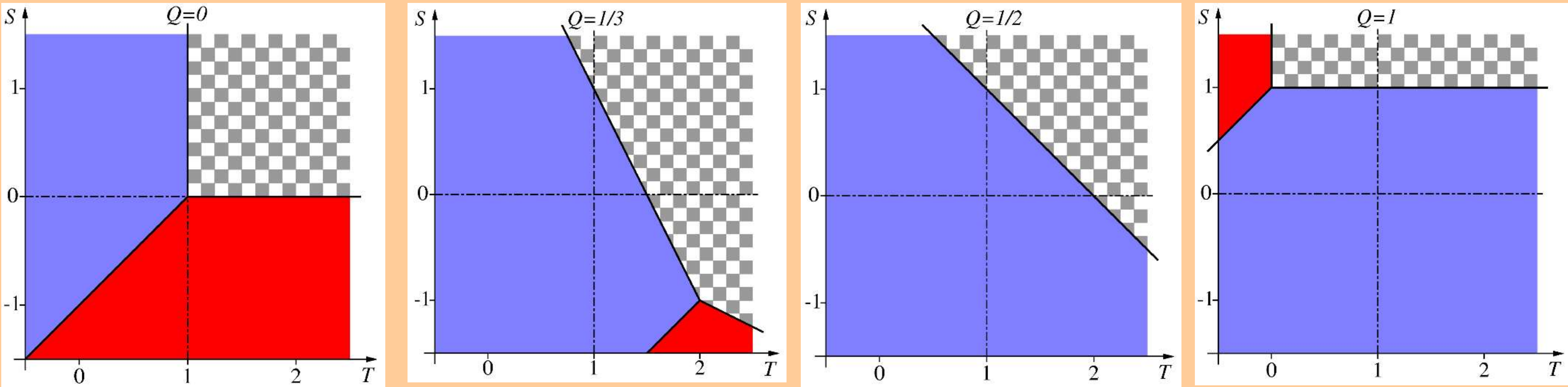
$U_y = 2[1 + S(1-Q) + QT]$,

C invasion, if $U_y > U_x$, or when

$$1 + S(1-2Q) > T(1-2Q)$$

It is equivalent to the prediction of risk dominance!

Phase diagrams at $K \to 0$ lovers' dilemma



The lovers' dilemma is described by the opening sentences of the paper by N. Fröhlich: Self-interest or altruism, what difference. *J. Conflict Resolution* 18, (1974) 55–73.

"There is a famous story written by O. Henry about a poor young couple in love at Christmas time ("The Gift of the Magi"). Neither has any money with which to buy the other a present, although each knows what the other wants. Each has only one prized possession: he, his father's gold pocket watch; she, her beautiful long hair. He has long been coveting a gold watch fob, while she has long admired a pair of tortoise shell hair combs in a nearby shop. The conclusion of the story is the exchange of gifts, along with a description of their means of getting the money for their purchases. He gives her the combs (having pawned his watch to raise the money). She gives him the watch fob (having cut and sold her hair)."

Coevolution of strategies and dynamical rules (K_x)

On a square lattice, player x with strategy s_x can imitate both one of her neighbours' strategy (s_y) and also their imitation rule, a Fermi–Dirac function characterised by K_y.

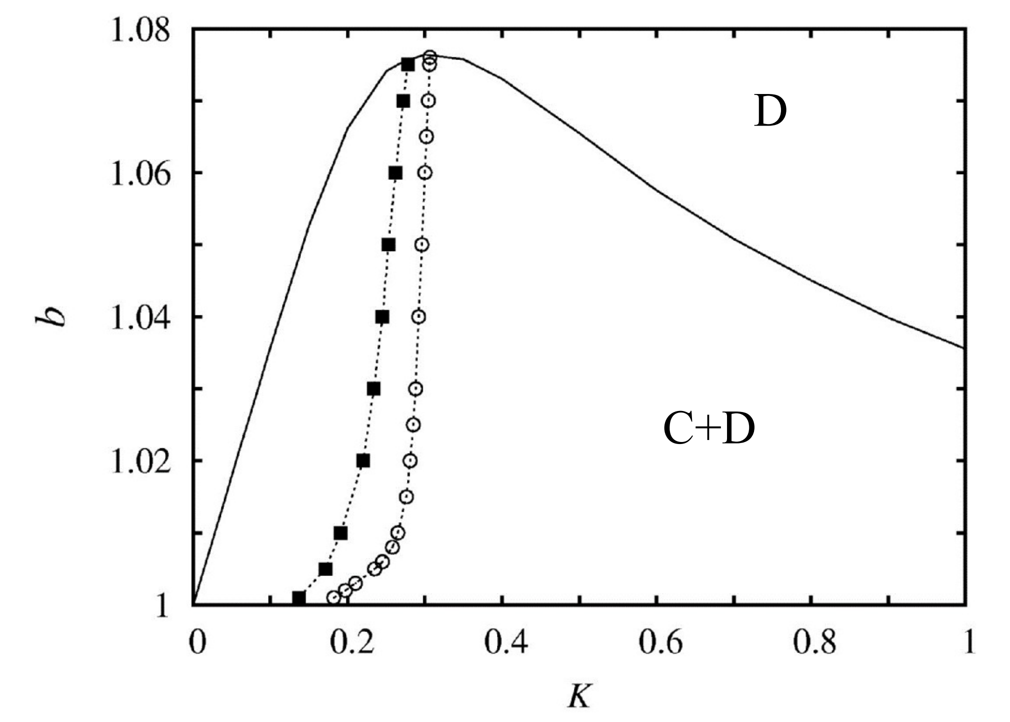
Initially (at t=0) s_x=C or D, and $K_x \in (K_{min},K_{max})$ chosen at random.

During the imitation process, player x can adopt in two consecutive steps the strategy s_y and K_y.

Simulations for $T= b$ and R=1, S=P=0:

In the final state, a single K_x (squares) is followed in the coexistence region.

The winning K_x are close to those K that optimize the average payoff (circles).



The winner may be K_x=0.

For example on the kagome lattice.

In the homogeneous C (or D) state the evolution of the K_x distribution is described by the voter model.

Summary of mechanisms supporting cooperation in PD games:

- punishing individual strategies (e.g. TFT)
- institutional punishment, laws, ethics, shut out
- fraternity
- additional options, e.g., multistrategy games (*C*+*D*+*L*, *C*+*D*+TFT, etc.)
- tagged players (e.g., family, group, tribe, mafia, etc)
  - group selection favours cooperation
- fixed connectivity structure for repeated games
  - it supports the formation of *C* colonies
  - suitable topology enhances the level of cooperation (overlapping triangles)
    - suitable noise level for several structures
  - inhomogeneity in the degree distribution
  - inhomogeneities in the strategy adoption
- destroying connections (knowing when to walk away)
- separation of learning and interaction graphs,
- …

Home assignments

10.1. Show that cooperators become extinct on the one-dimensional chain with nearest neighbour interactions for the evolutionary rule defined on page 1 when the payoffs are $1<T<2$ and $S=0$!

10.2. Reproduce the phase diagrams plotted on page 13 by evaluating the potential matrix and determining its maximal entry (as a function of T and S for $Q=0$, 1/3, 1/2, and 1) that selects the preferred Nash equilibrium!